

$$E(x) = -\frac{1}{2} \sum_i \sum_j w_{ij} x_i x_j$$

En efecto, sea k, l pareja de índices de $E(x)$. Podemos escribir:

Goles inner Energy

On the occasion of 70's birthday Goles' Festschrift

$$E(x) = -\alpha_k \sum_j w_{kj} x_j - x_l \sum_j w_{lj} x_j + \text{resto}$$

Sergio Rica, UAI

July, 14th 2021

$$x' = (x_1, \dots, x_k, \dots, x_l, \dots, x_n)$$

Plan

- Goles as a teacher
- Mr Goles goes to "la Moneda"
- Goles as a colleague
- Goles energy principle
- Finale

Consideremos $G=(V,E)$ grafo finito
 $(|V|=n)$, no orientado y simétrico $(i,j) \in E \Leftrightarrow (j,i) \in E$
 Sea $W=(w_{ij})$ la matriz de interacción

$$w_{ij} = 1 \Leftrightarrow (i,j) \in E.$$

$$w_{ii} = 0.$$

Para nuestra aplicación consideramos
 $|V_i| = \# \text{ vecinos}$ $V_i = \{j \in V / (i,j) \in E\}$ = cpto de vecinos

1. Modelo de segregación

(1) 25

Asumo
 individuos
 M (ambos)

Consideremos $G=(V,E)$ grafo finito de n nodos
 $(|V|=n)$, no orientado y simétrico $(i,j) \in E \Leftrightarrow (j,i) \in E$
 Sea $W=(w_{ij})$ la matriz de interacción

$$w_{ij} = 1 \Leftrightarrow (i,j) \in E.$$

$$w_{ii} = 0.$$

Para nuestra aplicación consideramos
 $|V_i| = \# \text{ par}$ $V_i = \{j \in V / (i,j) \in E\}$ = cpto de vecinos

Asumamos que sobre cada nodo hay un
 individuo de color 1 o -1 y definamos
 M (ambos) de satisfacción $\theta_i \geq \frac{|V_i|+1}{2}$

Dinámica.

Si $\exists (k,l) \in E$ tales que
 $\sigma_k = -\sigma_l$ y además ambos
 están insatisfechos:

25

más de θ_i vecinos

Goles as a Teacher

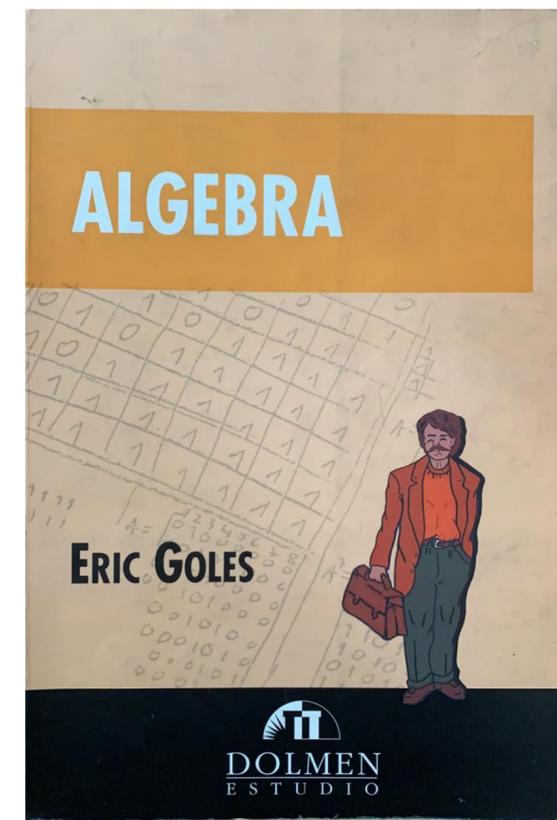
- I met Goles on August 1984, during his lectures on linear Algebra at the School of Engineering at the U of Chile.



- Always in a bad mood... he disliked the minimal noise. Always smoking five cigarettes per lecture.
- “ $n + 1$ vectors in a space of dimension n cannot be linearly independent”
- How to multiply 2×2 matrices with seven multiplications....
- Eigenvalues... & Tacoma narrow

At Universidad Adolfo Ibanez

- “Professor can you repeat what did you say ?...”
- The foundation of the PhD in Complex Systems at UAI



Mr. Goles goes to “La Moneda”

- In 1985, during the Chilean fascism dictatorship, Goles was one of the leaders of the resistance against an imposed non-Academic Dean named Poblete.
- To be or not to be...
- But, 15 years later, he went to politics.... under Lagos Presidency he became the President of the Chilean Research Agency, etc.
- “I was seduced by political power: to meet Chirac, Spanish King...”
- But he came back to Science.
- To be or not to be...

Goles as a Colleague

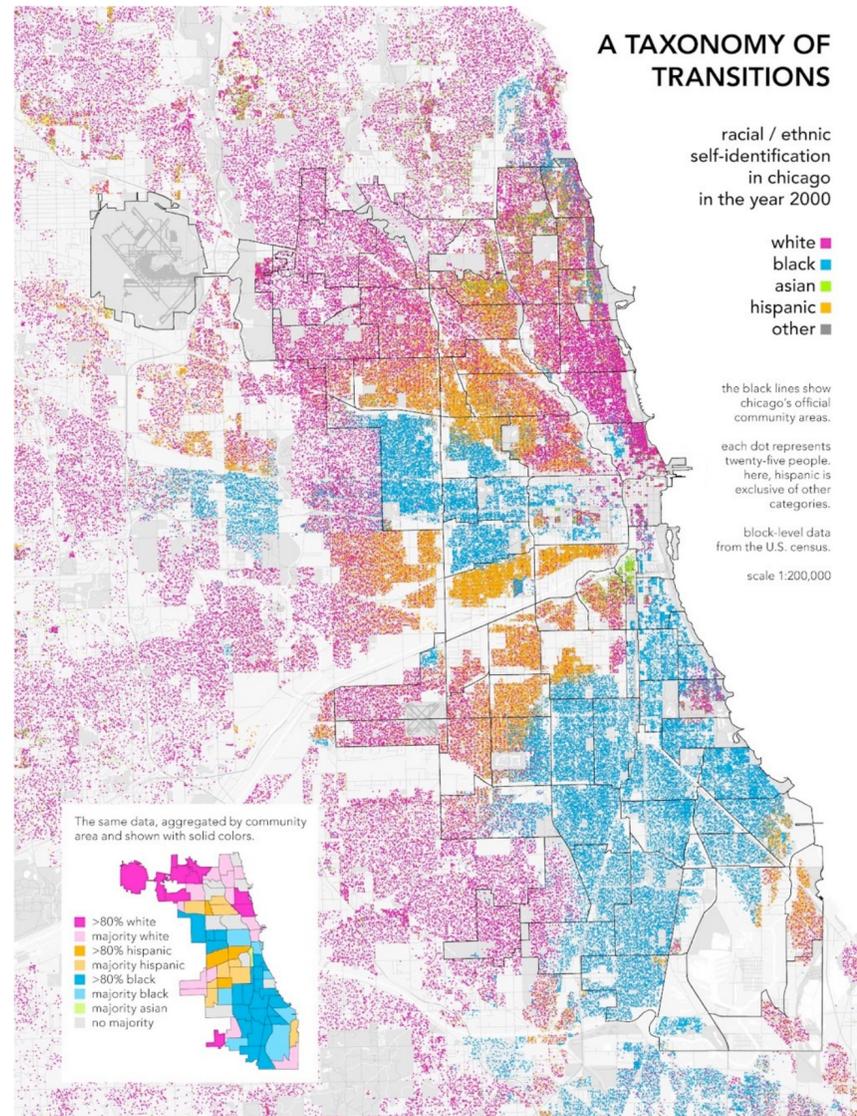
The UAI years

- I joined the UAI in 2009
- Enthusiastic, motivated, happy doing science
- Intellectually broadly active
- He loved to be at the cafe or dinning room....
- He is the best talking about Goles... (perhaps in excess, but he is a funny man)
- Perhaps a bit insistent....
- Schelling Segregation model, and others



Goles energy principle

Schelling Segregation model



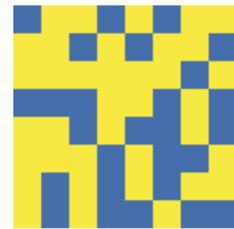
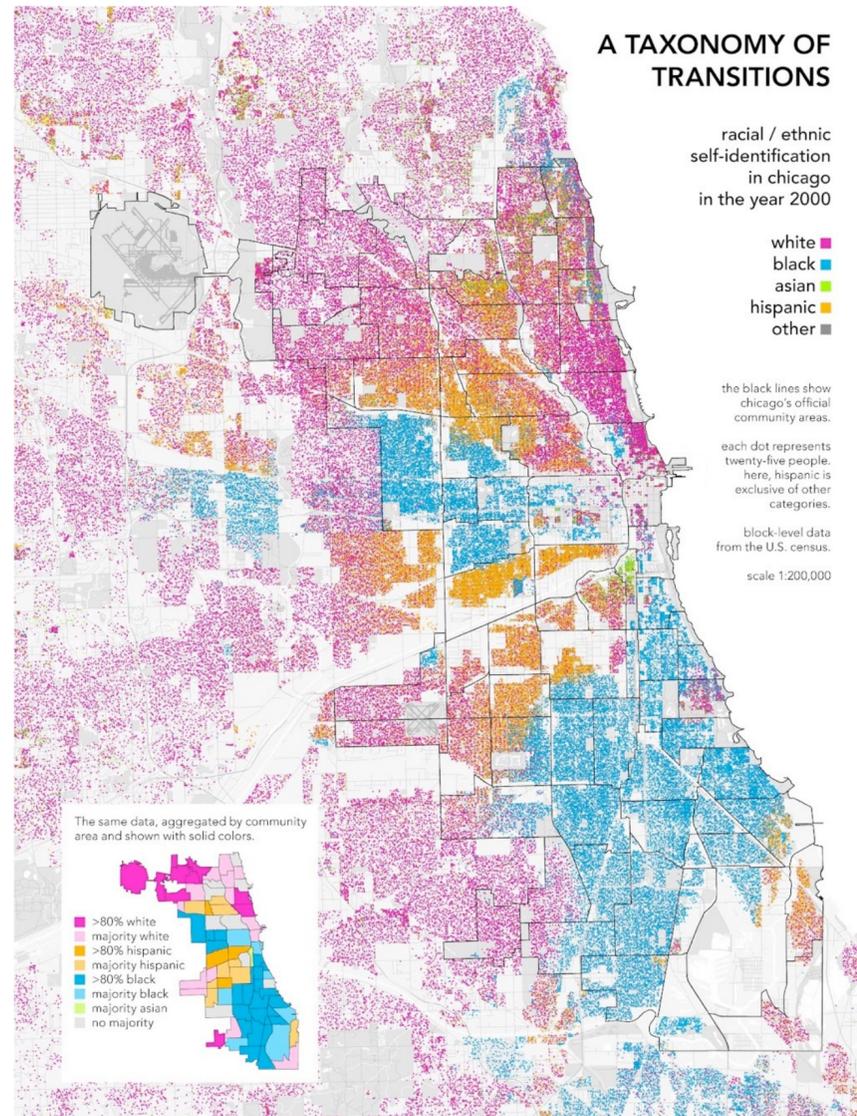
After Bill Ranking (Instagram)

Goles energy principle

Schelling Segregation model

$$x_k = \pm 1$$

An individual is unhappy if there are more than θ individuals of the other type in its neighborhood.

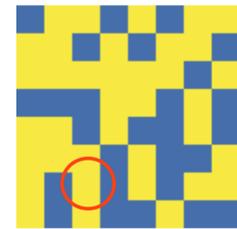
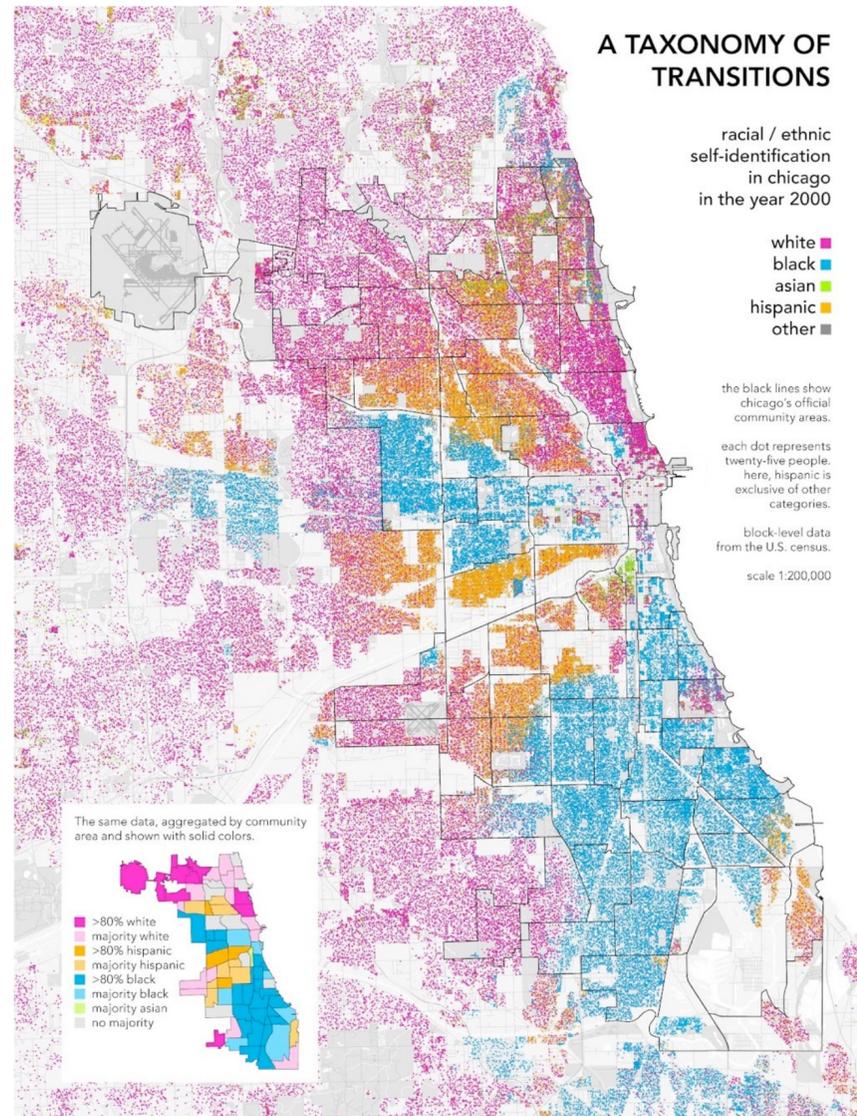


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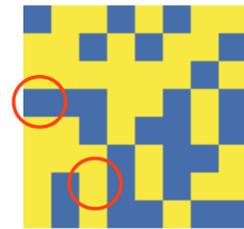
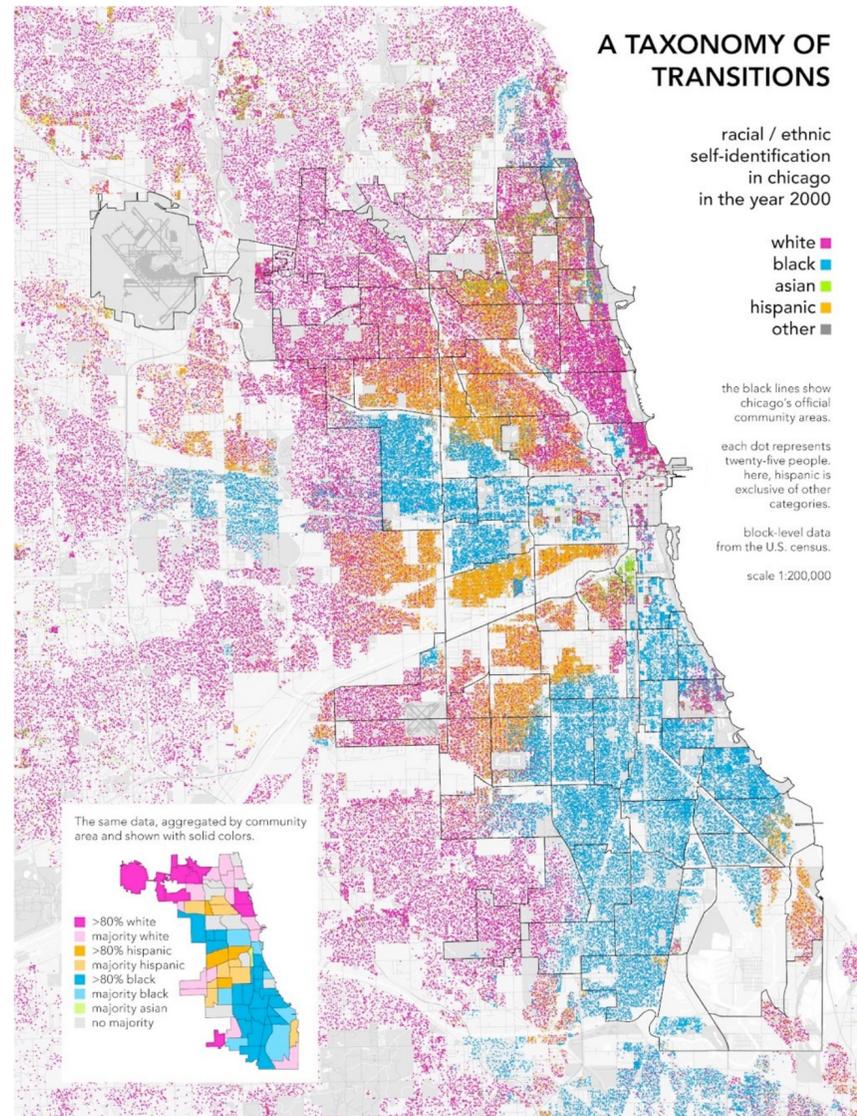


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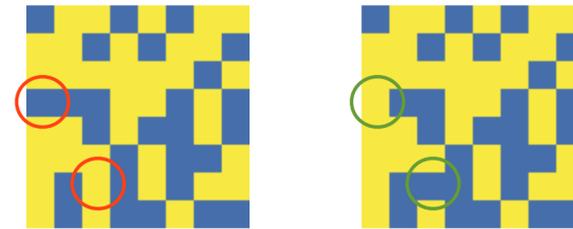
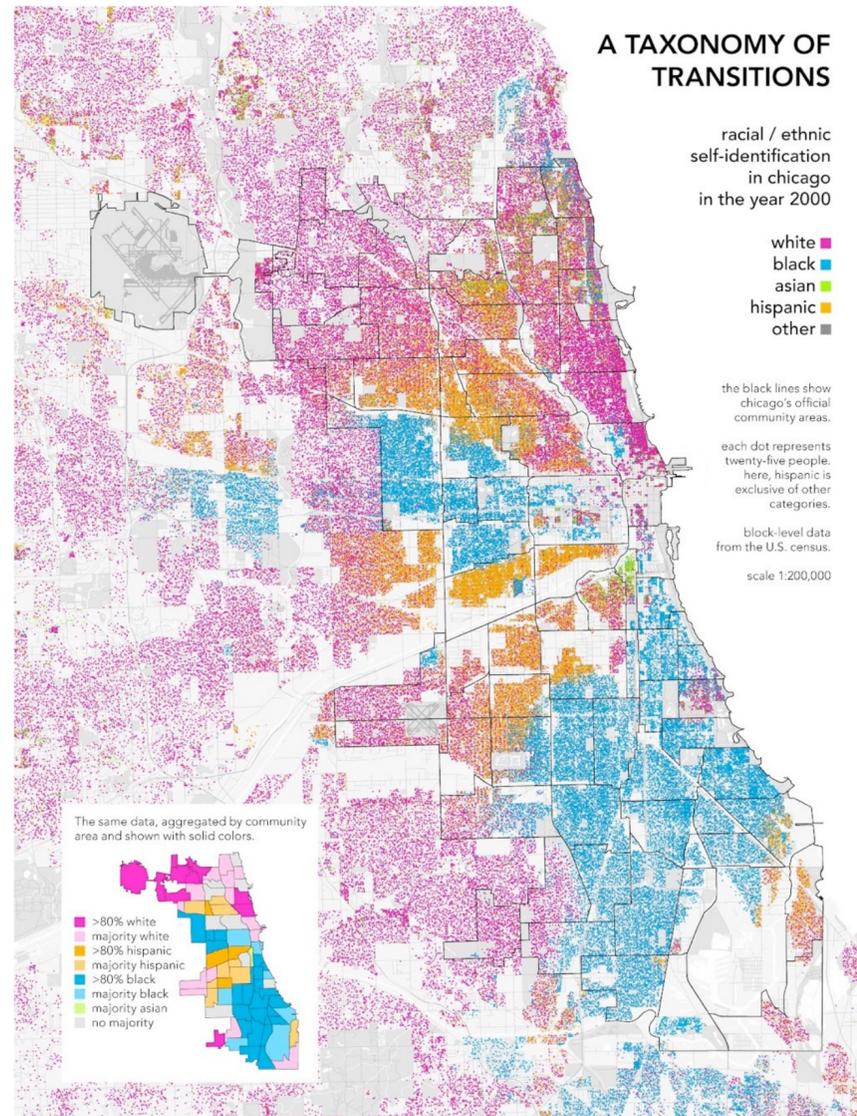


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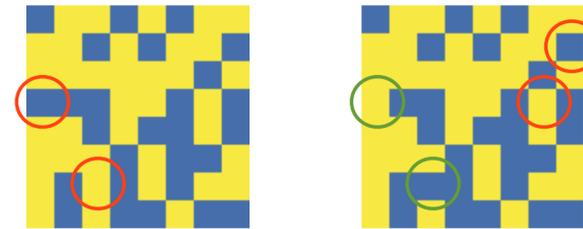
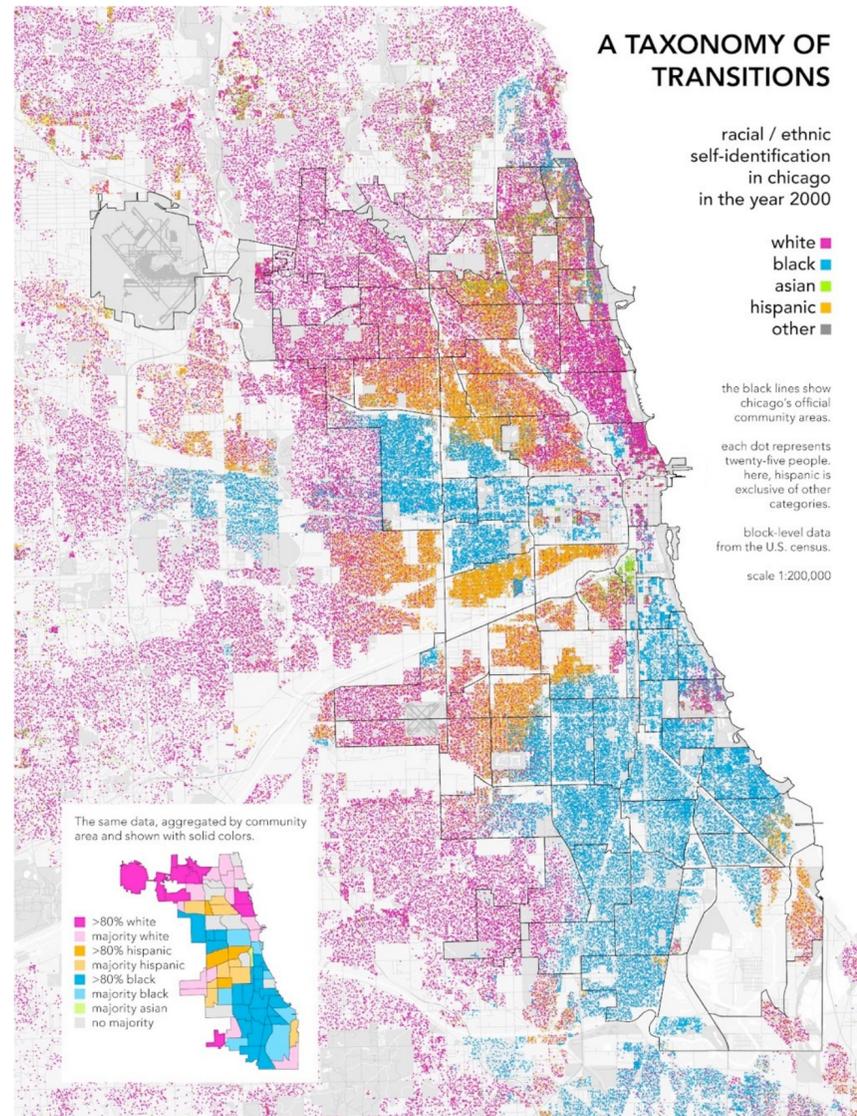


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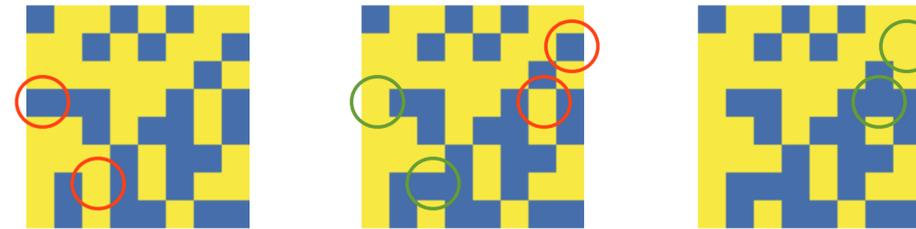
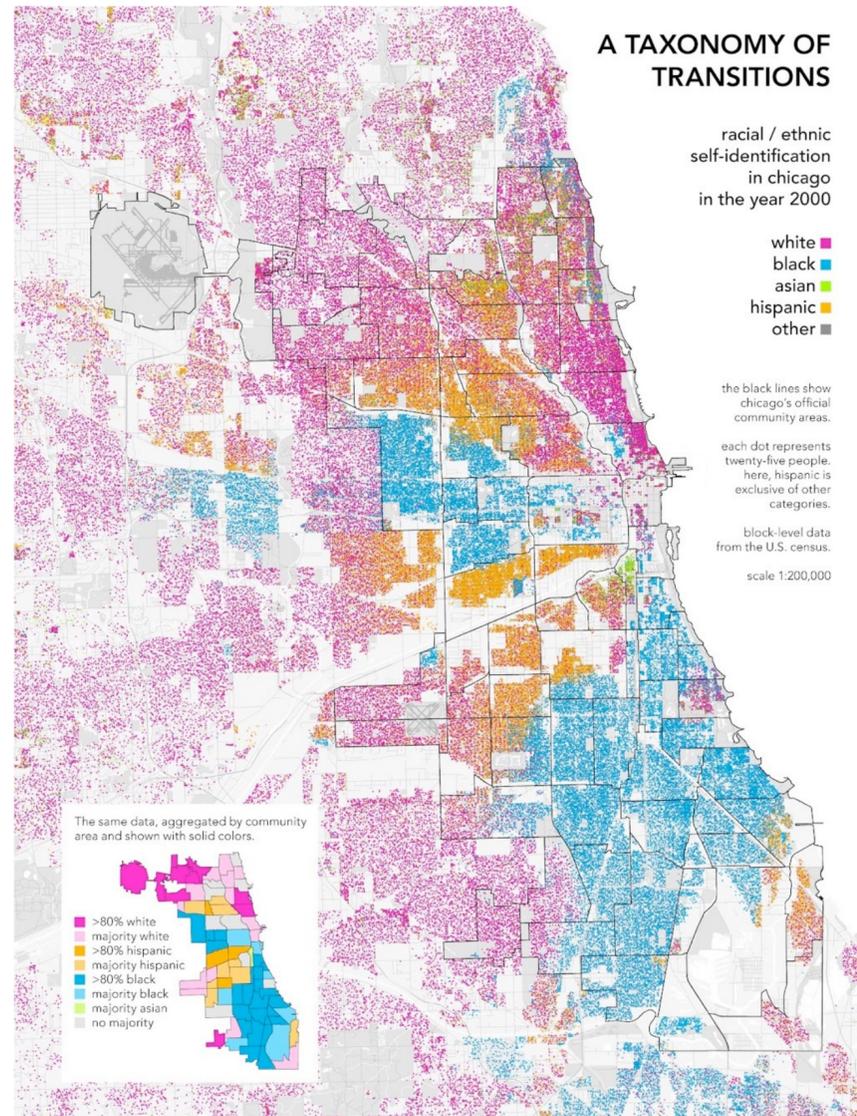


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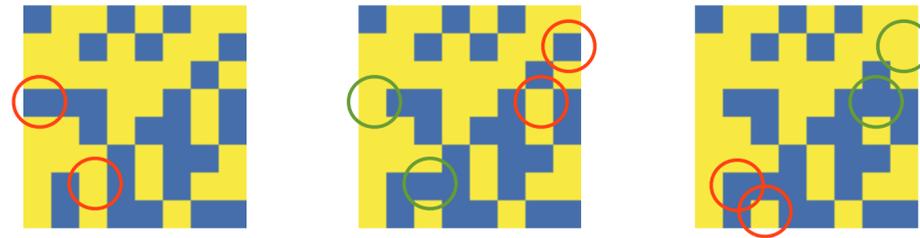
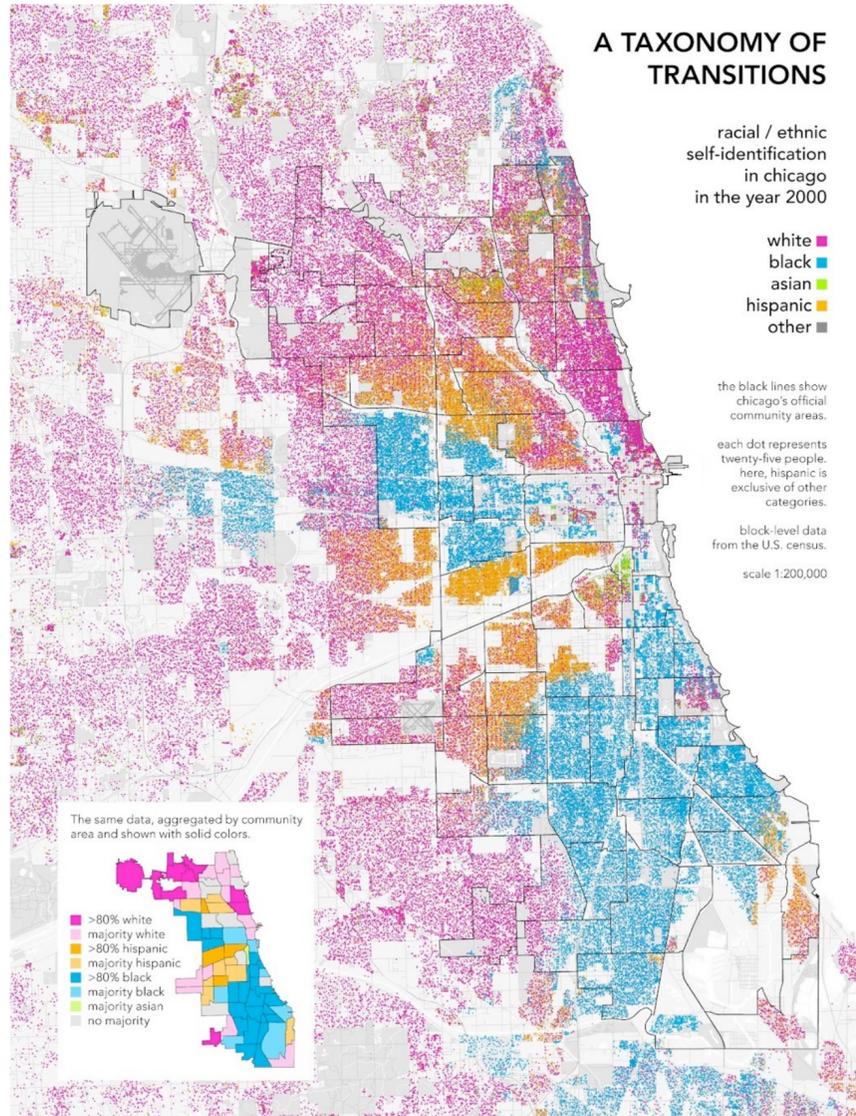


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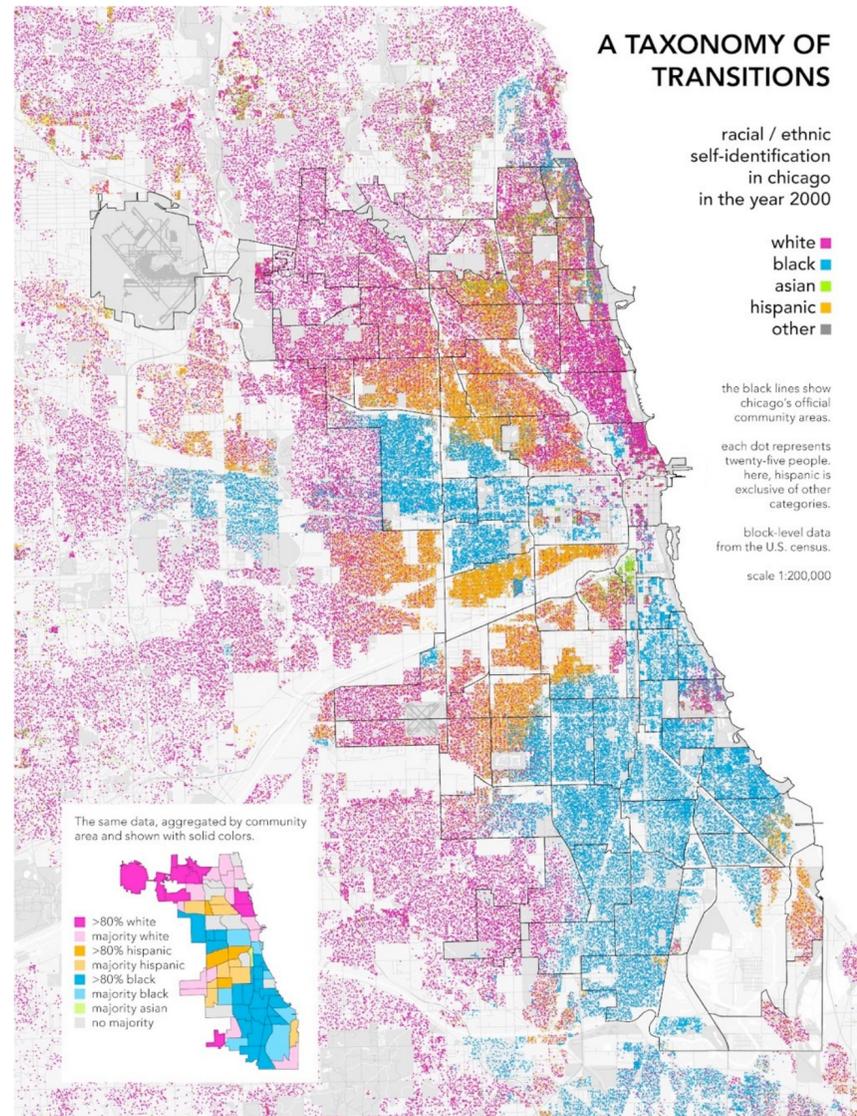
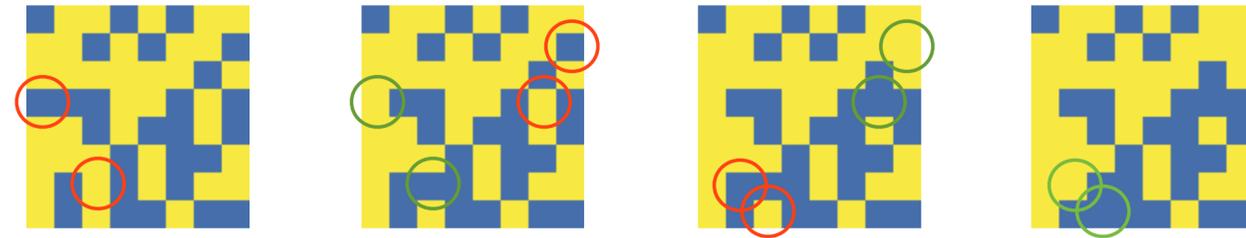


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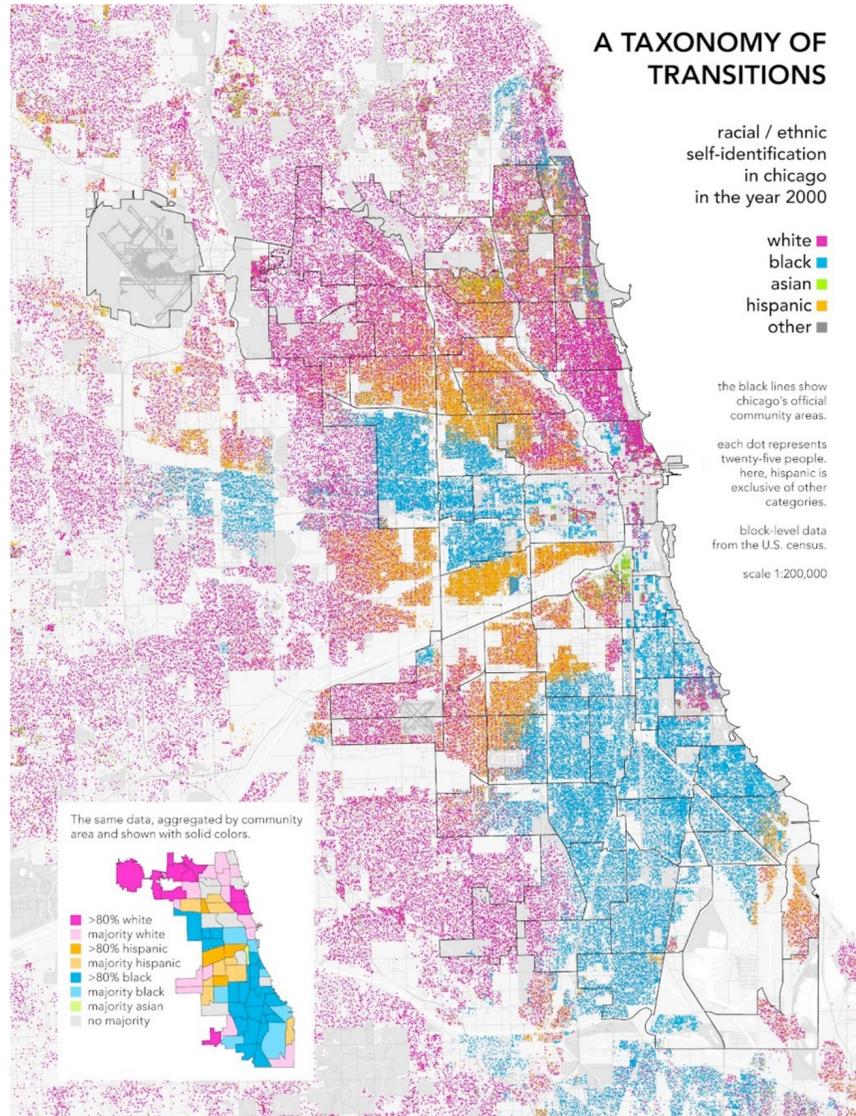
After Bill Ranking (Instagram)

Goles energy principle

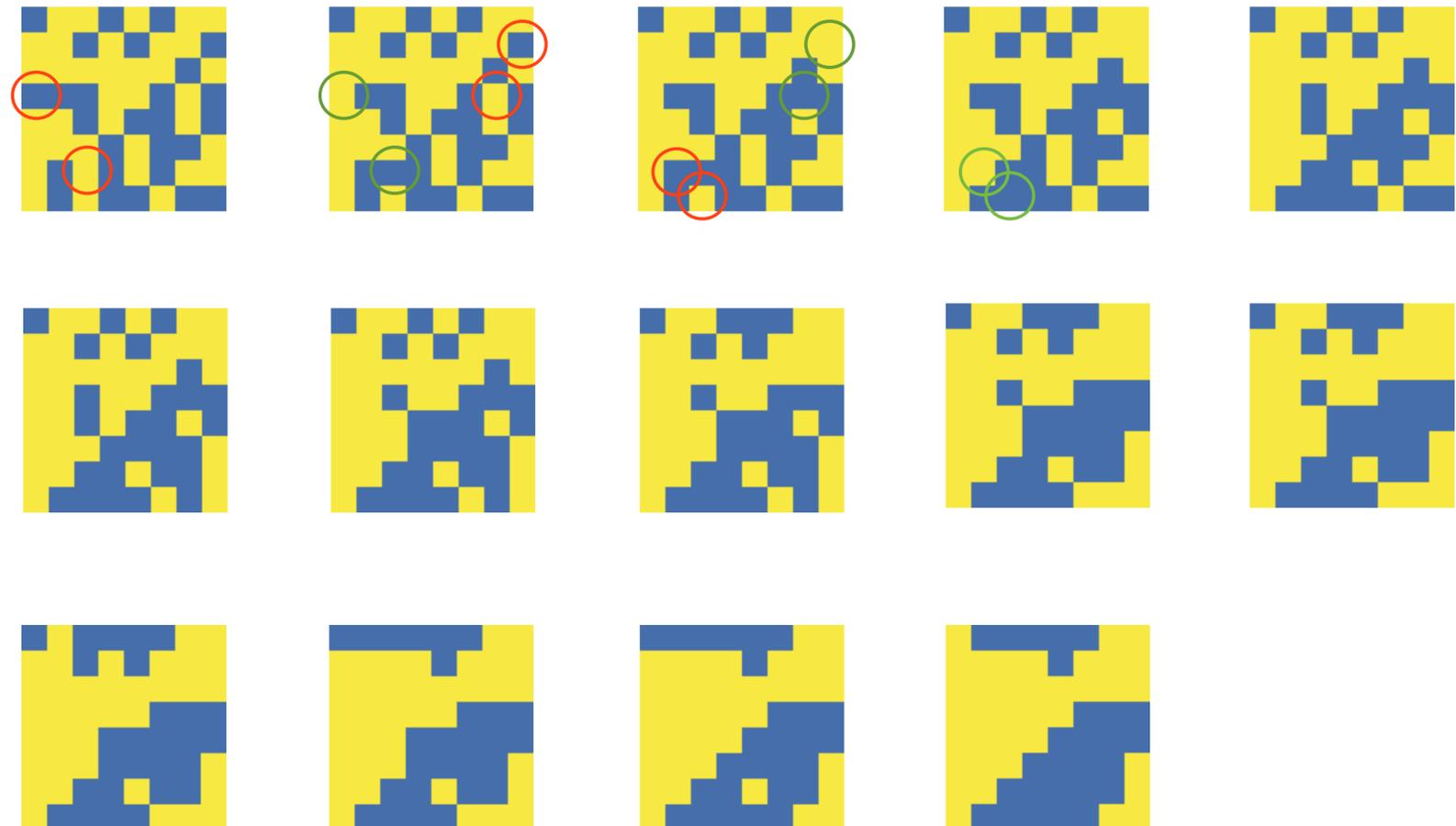
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After Bill Ranking (Instagram)



Goles energy principle

- In 1980-1981 he published a series papers with J Olivos on iterations on threshold function and his 3rd cycle thesis Dr Ing. Université de Grenoble.
- In SIAM J. Alg. Disc. Meth (1982) provides a sufficient condition for certain iterations (*block Gauss-Seidel*) have only fixed points, and there is an application on spins.
- (1983) Transient Length...
- (1985) Decreasing energy fu

Discrete Applied Mathematics 6 (1983) 95-98
North-Holland Publishing Company

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The *energy* associated with the sequential iteration on F is the function $E: \{0, 1\}^n \rightarrow \mathbb{R}$, defined by:

$$\forall x \in \{0, 1\}^n, \quad E(x) = -\frac{1}{2} \sum_{i=1}^n x_i \sum_{j \neq i} a_{ij} x_j + \sum_{i=1}^n (\theta_i - a_{ii}) x_i.$$

For any trajectory $(x(t))_{t \geq 0}$ we will denote $E(t) = E(x(t))$.

2. A general bound on $T(F)$

Lemma 1. *If $A = (a_{ij})$ is a symmetric matrix with non-negative diagonal elements, then:*

$$x(t+1) \neq x(t) \Rightarrow E(t+1) < E(t).$$

COMMUNICATION

TRANSIENT LENGTH IN SEQUENTIAL ITERATION OF THRESHOLD FUNCTIONS

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Discrete Applied Mathematics 12 (1985) 261-277
North-Holland

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DECREASING ENERGY FUNCTIONS AS A TOOL FOR STUDYING THRESHOLD NETWORKS*

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Françoise FOGELMAN-SOULIE

LDR, c/o CESTA 1 rue Descartes, 75005 Paris, and Univ. of Paris V, France

Didier PELLEGRIN

IMAG, TIM3, Grenoble, France

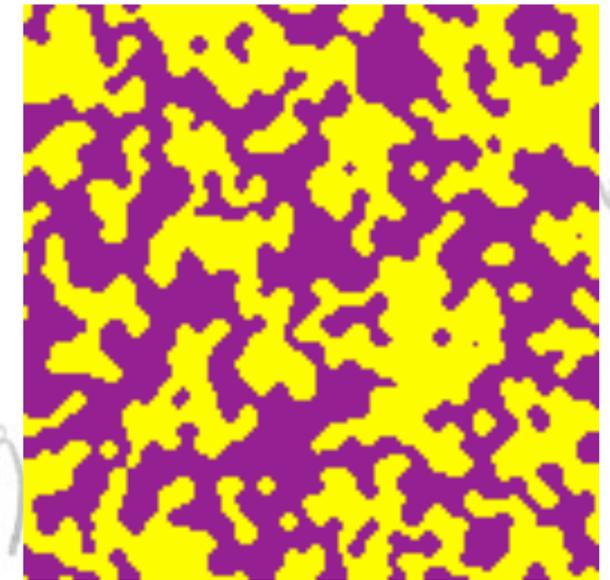
Received 24 May 1984

Revised 25 June 1985

Block sequential iterations of threshold networks are studied through the use of a monotonic operator, analogous to the spin glass energy. This allows to characterize the dynamics: transient and fixed points. We then extend this method to networks of generalized majority functions and spin glasses.

Schelling segregation model.

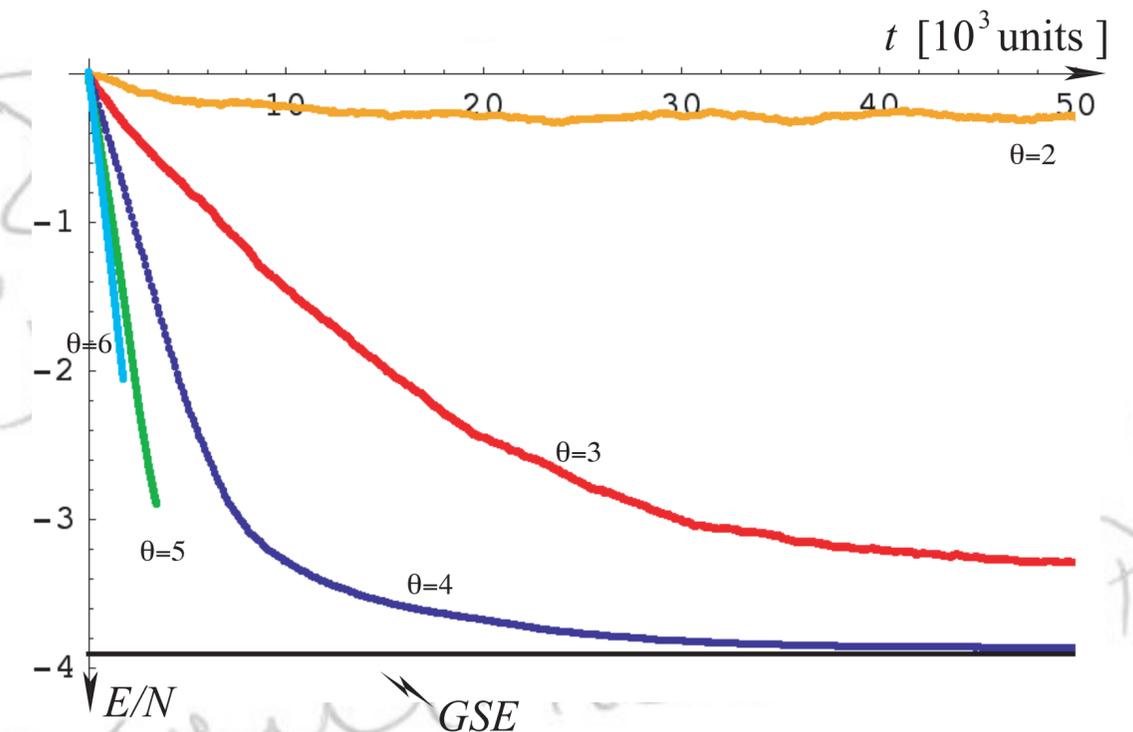
If $\theta > 5$, then “energy”, $E = -\frac{1}{2} \sum_{i \neq k} J_{ik} x_i x_k$, decreases strictly during the evolution.



$\theta = 5$



$\theta = 3$

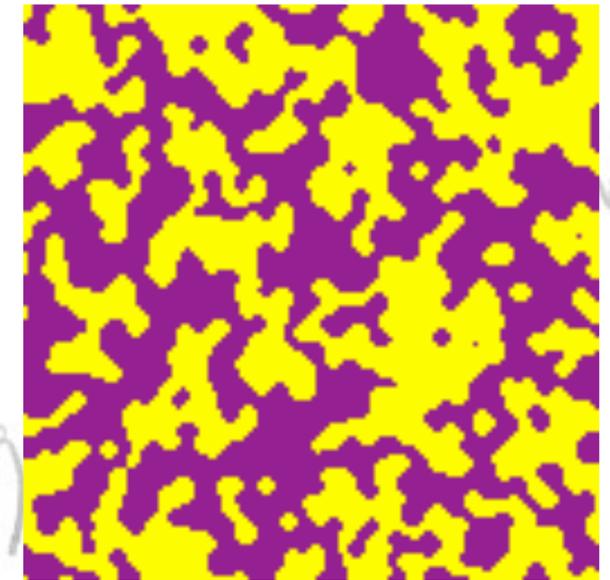


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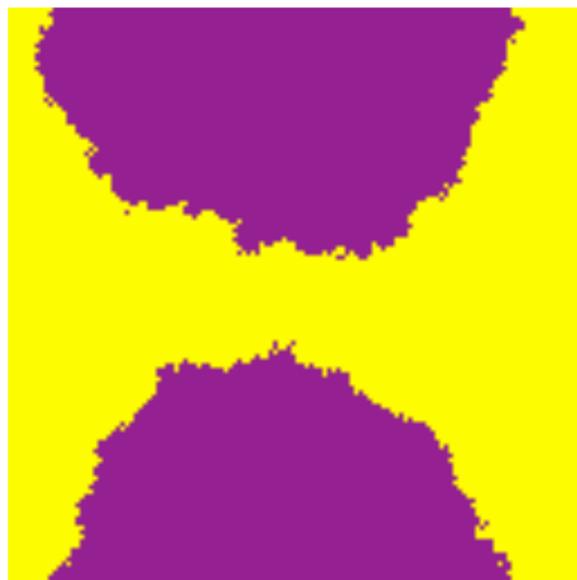
If $\theta > 5$, then “energy”, $E = -\frac{1}{2} \sum_{i \neq k} J_{ik} x_i x_k$, decreases strictly during the evolution.

$$E = -4N + 2 \times \left(3 \sum \text{edges} - \sum \text{corners} \right)$$

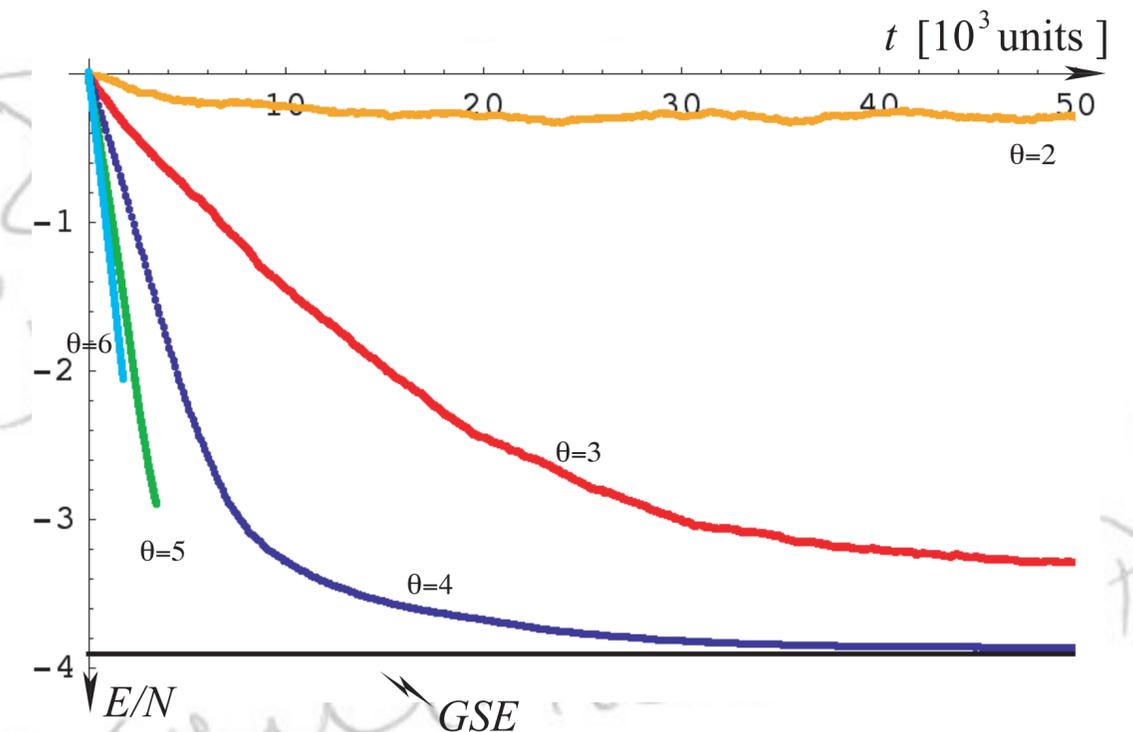
$$= -4N + 2 \times (3 \times \text{perimeter} - \text{Nb. of corners}),$$



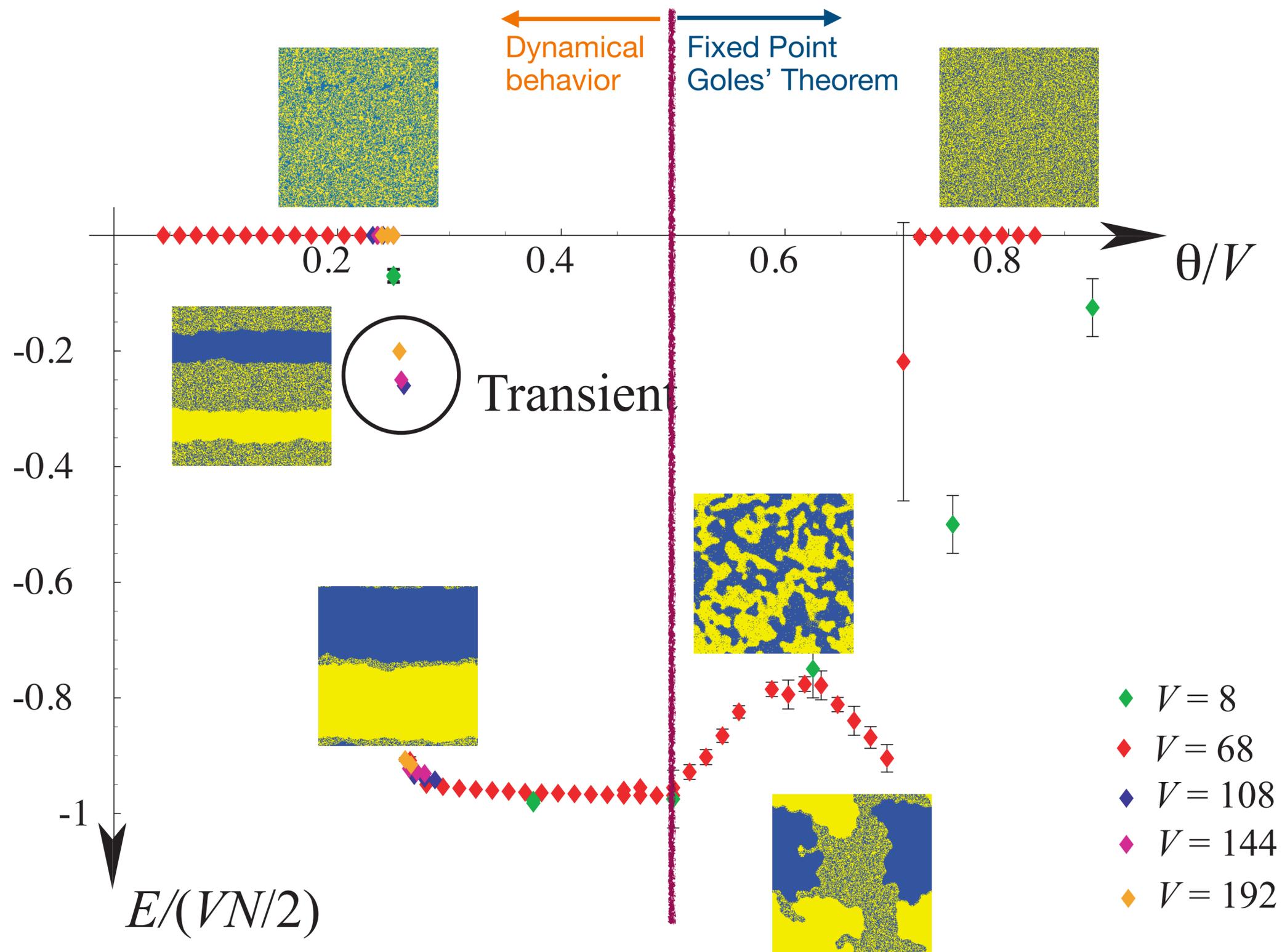
$\theta = 5$



$\theta = 3$



The energy and phase transitions



The energy in physics

- Thermodynamical potentials are used since Maxwell, Boltzmann and Gibbs
- Ising energy plays an important role in phase transitions in statistical physics.
- Energy concept goes back to Huygens...
- After Lagrange, Hamilton the energy plays a central role in physics. It survives along the clouds and storms seasons in physics

$$E(x) = -\frac{1}{2} \sum_i \sum_j w_{ij} x_i x_j$$

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But energy is conserved !

$$E(x) = -\frac{1}{2} \sum_i \sum_j w_{ij} x_i x_j$$

The Q2R model

Vichniac 1984

- Lattice : *2D periodic square*
- State $x_k = \pm 1$
- Vicinity $\Phi_k(u) = \begin{cases} -1 & \text{if } \sum_{i \in V_k} u_i = 0 \\ 1 & \text{if } \sum_{i \in V_k} u_i \neq 0 \end{cases}$
- The Rule $x^{t+1} = x^{t-1} \Phi(x^t)$
- Reversible $x^{t-1} = x^{t+1} \Phi(x^t)$

Energy Conservation

Pomeau 1984

$$\begin{aligned} y^{t+1} &= x^t \\ x^{t+1} &= y^t \Phi(x^t). \end{aligned}$$

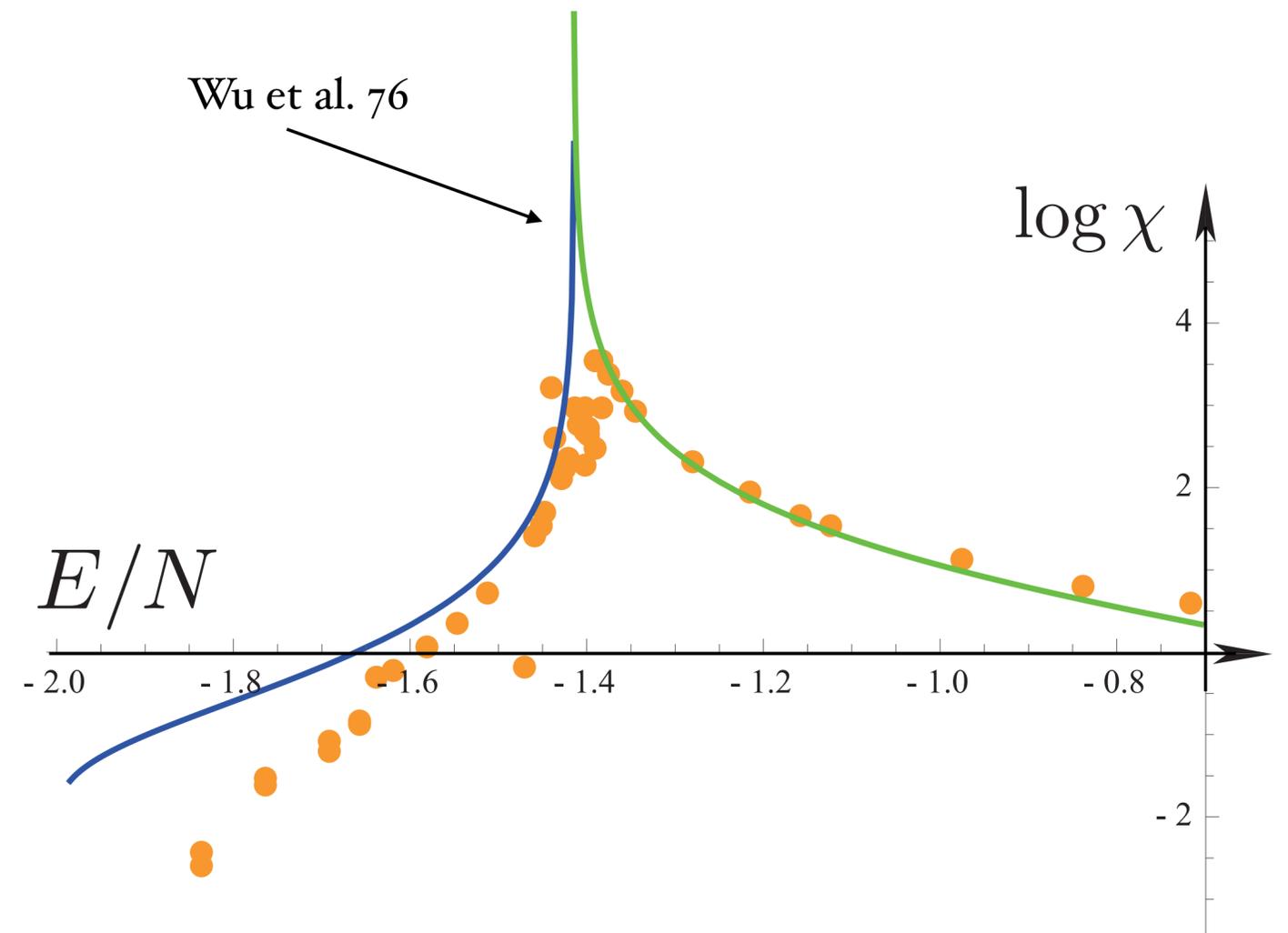
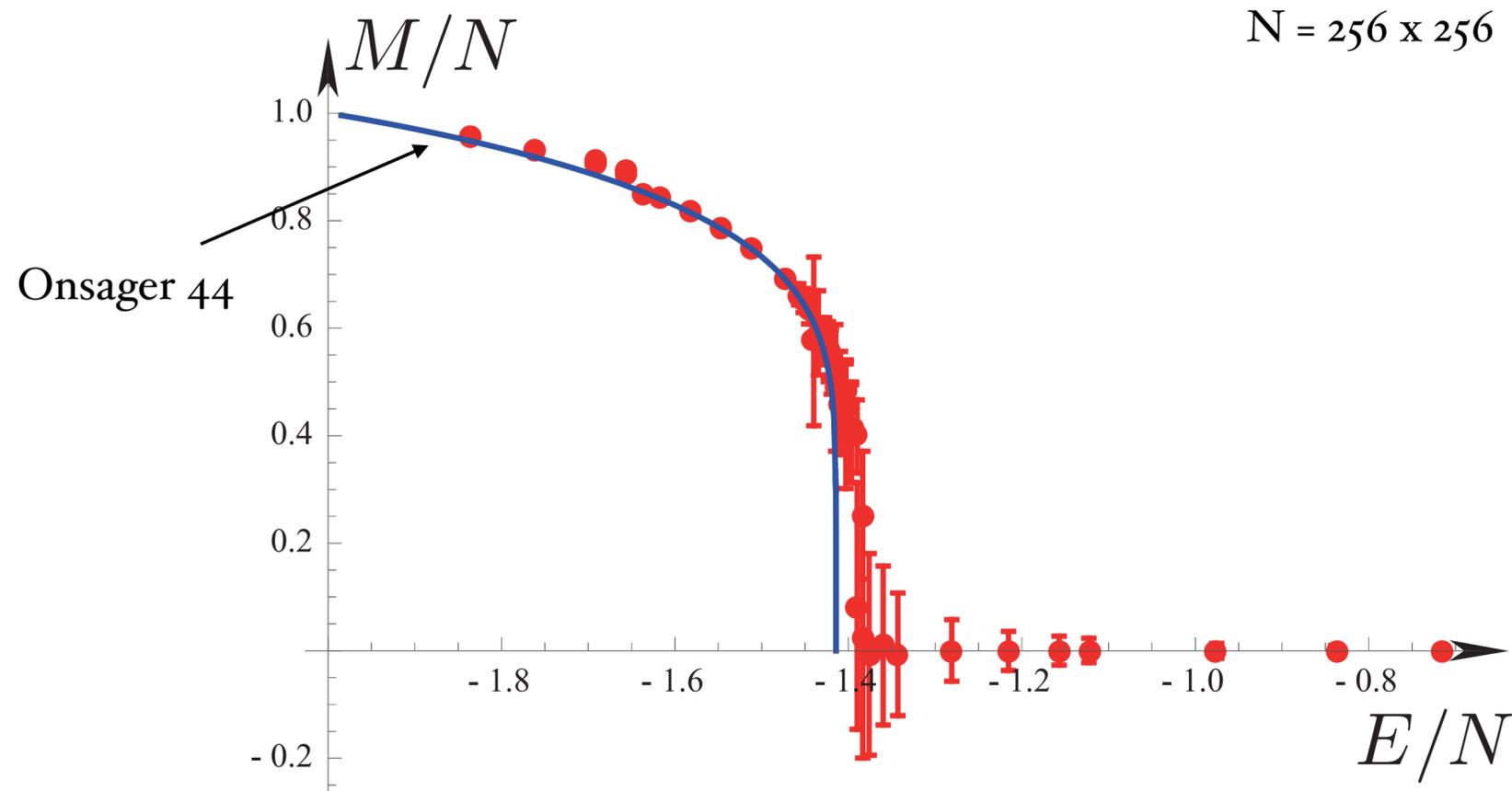
The following quantity

$$E[\{x^t, y^t\}] = -\frac{1}{2N} \sum_{\langle i, j \rangle} x_i^t y_j^t$$

is conserved by the evolution.

The phase space is compound by a few number of fixed points and a huge number of cycles, which periods may be exponentially long.

The Q2R model (Ising phase transition)



No adjustable parameters & Singular behavior in the limit $N \rightarrow \infty$. I would like to emphasize the possibility of unexpected singular behavior in discrete models.

Finale



Haydée :

Aunque sea inútil
valga este ejercicio
de retórica aplicada
como otro intento fallido
por creer en algo
por olvidar que no somos nada,
ni siquiera este lugar común.

Bien qu'il soit inutile
cet exercice vaut-il
de rhétorique appliquée
comme une autre tentative infructueuse
pour croire en quelque chose
pour oublier qu'on n'est rien,
même pas ce lieu commun

Although it is useless
is this exercise worth
of applied rhetoric
as another failed attempt
for believing in something
for forgetting that we are nothing,
not even this common place