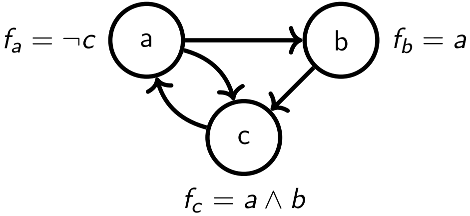
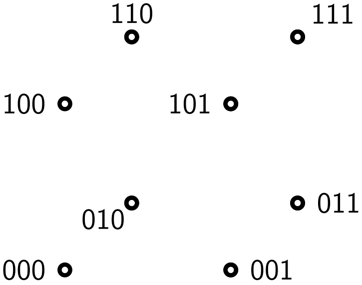
Associating parallel automata network dynamics and strictly one-way cellular automata

Pacôme Perrotin, Aix-Marseille University

Automata networks





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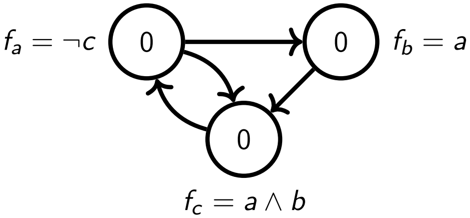

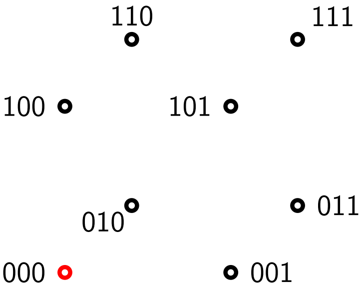

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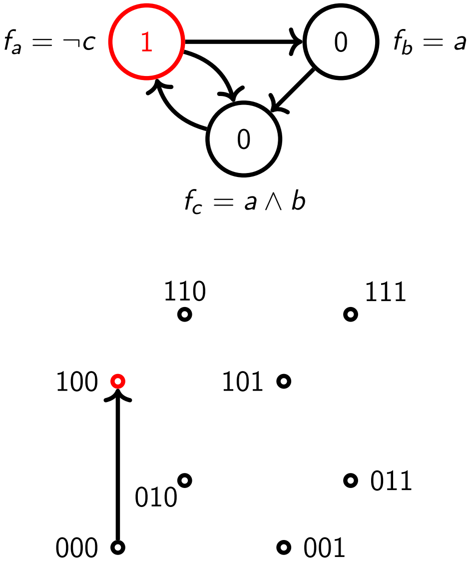

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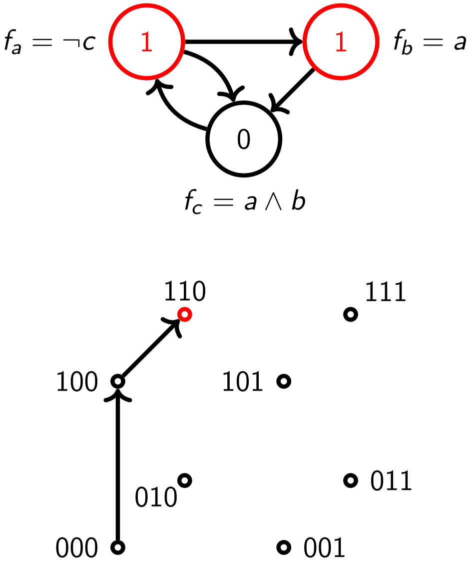

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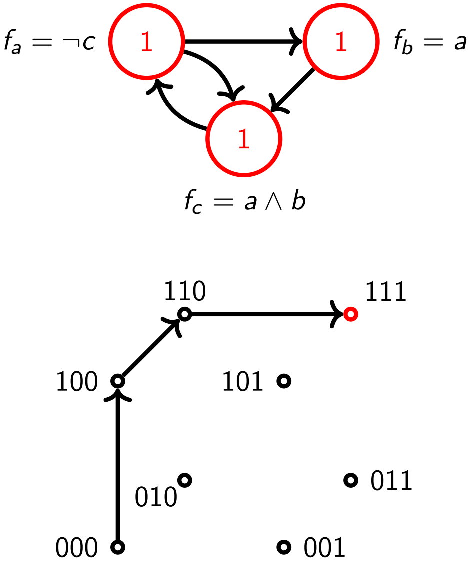

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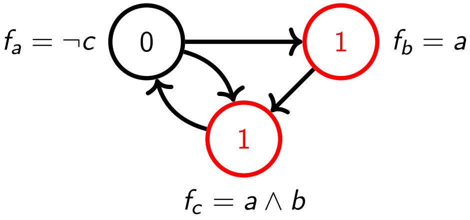

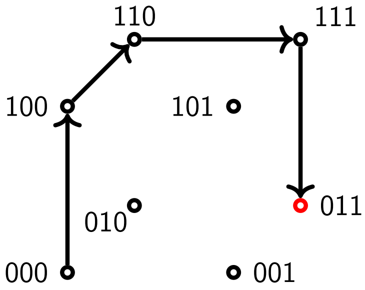

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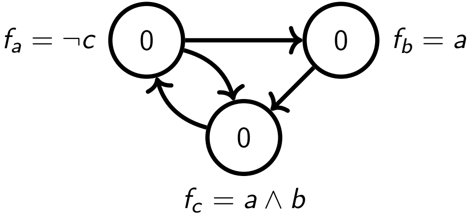

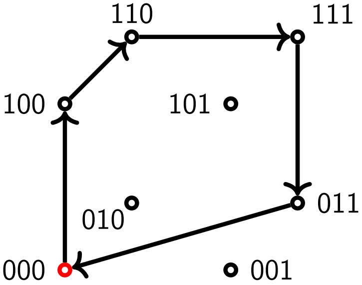

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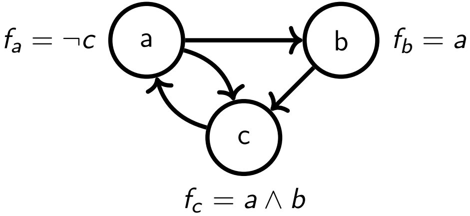
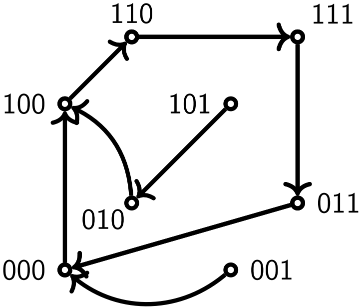
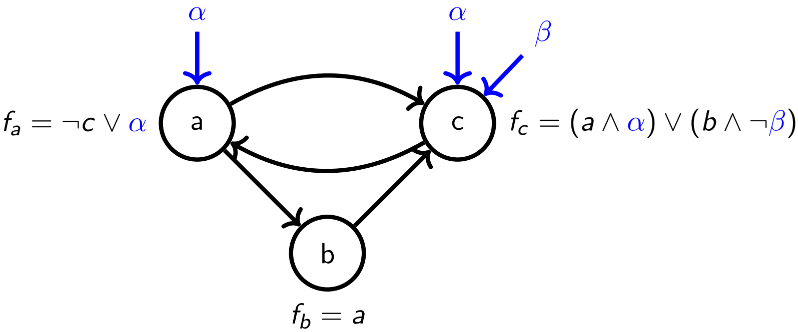


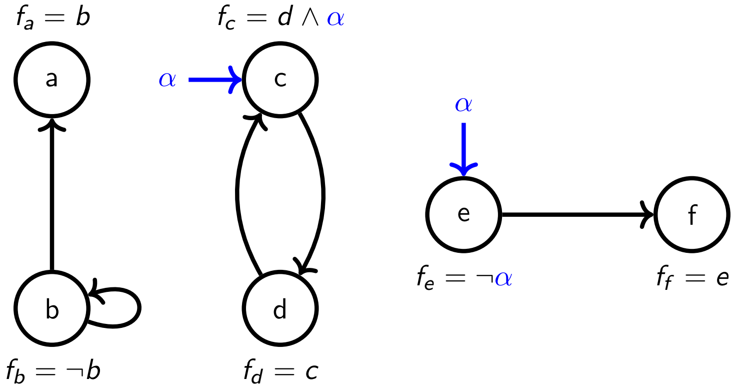


Modules

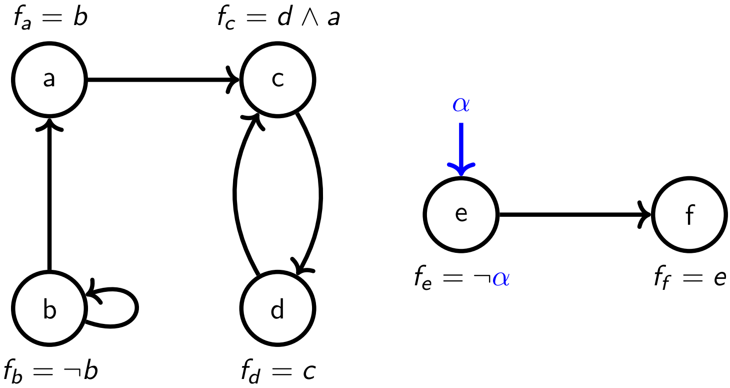


α
α
β
$f_a = \neg c \vee \alpha$
a
c
$f_c = (a \wedge \alpha) \vee (b \wedge \neg\beta)$
b
$f_b = a$

Wirings


$f_a = b$
a
$f_c = d \wedge \alpha$
α
c
α
e
f
$f_e = \neg\alpha$
$f_f = e$
b
d
$f_b = \neg b$
$f_d = c$

Wirings



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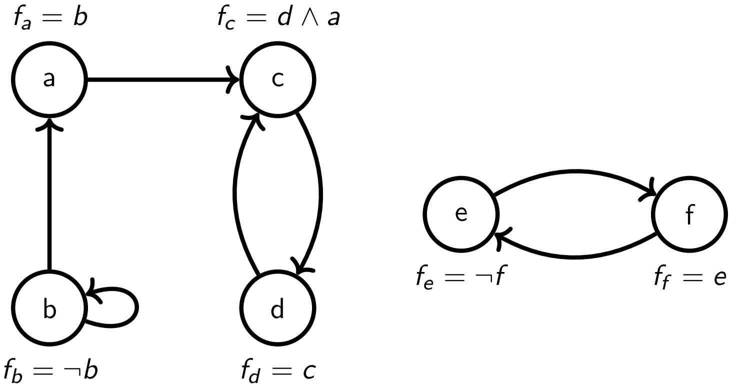

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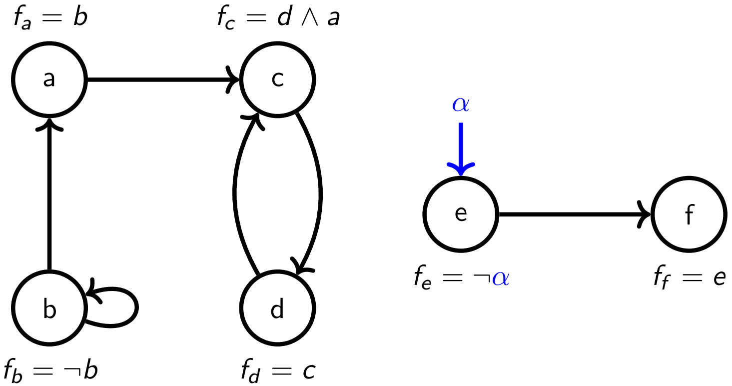

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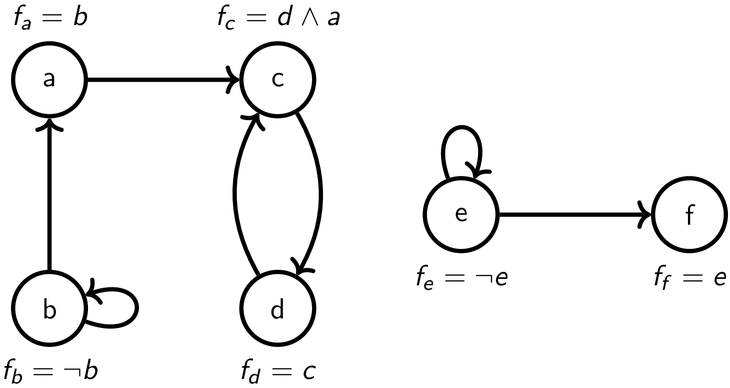
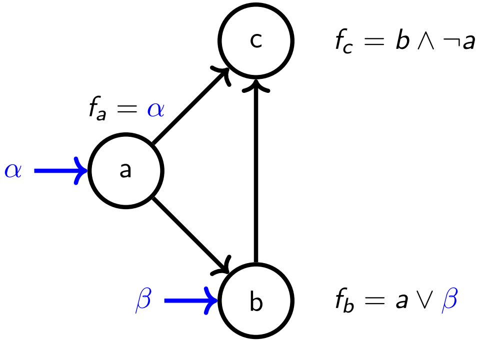


Acyclic networks and output functions



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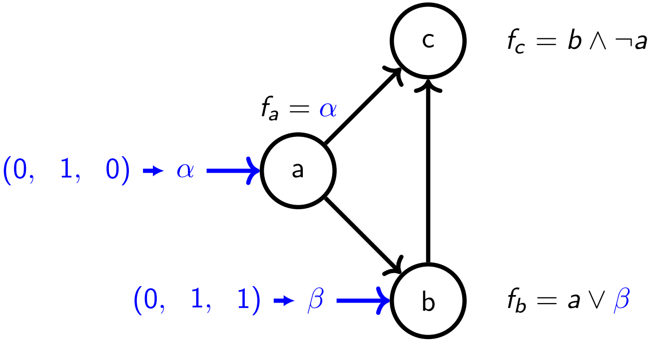

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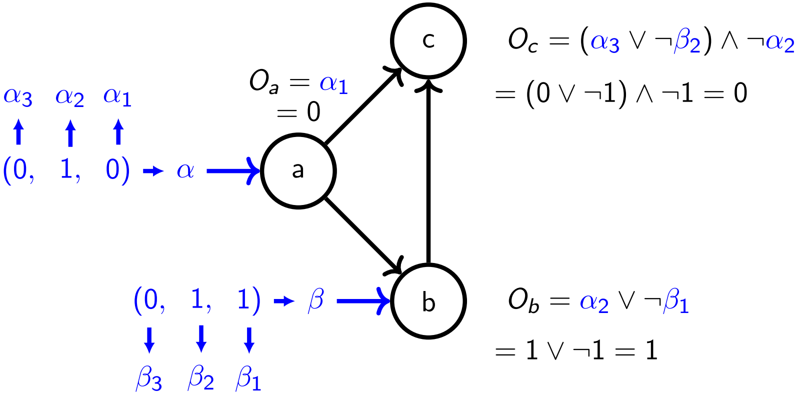

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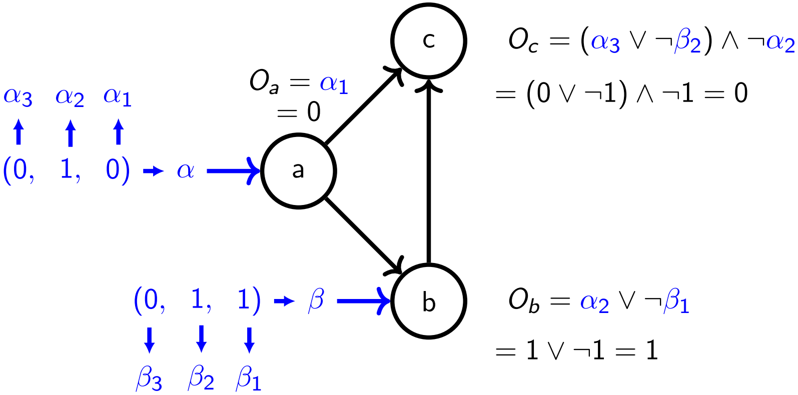


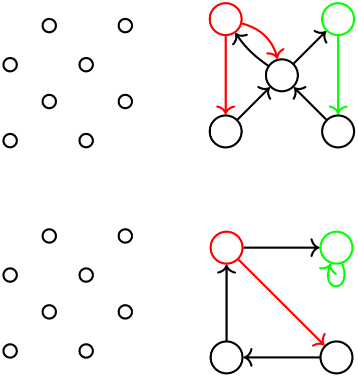
Theorem (Perrot, Perrotin, Sené 2010)

If two acyclic modules have equivalent output functions, then wiring them symmetrically will result in two networks with isomorphic attractors.

Sketch of proof

Theorem (Perrot, Perrotin, Sené 2020)

If two automata networks have equivalent output functions on two respective feedback vertex sets, then they have isomorphic limit dynamics.



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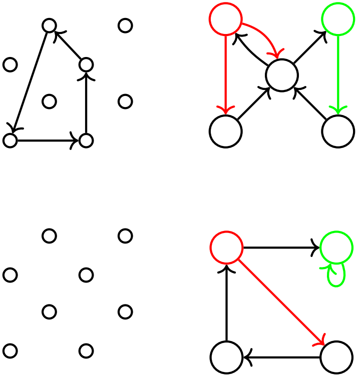

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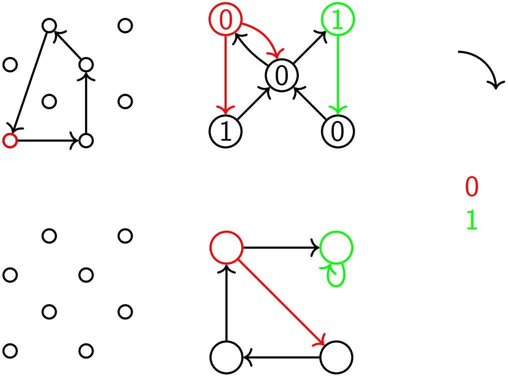

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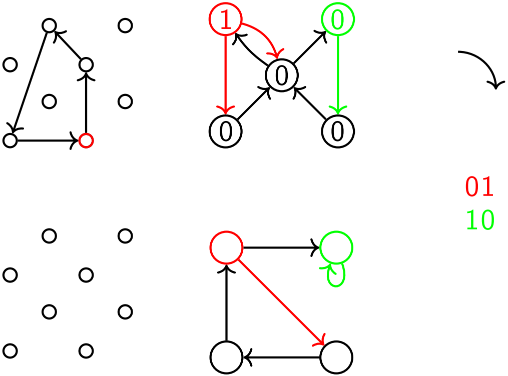

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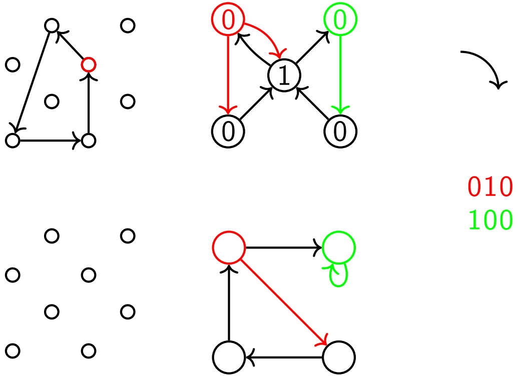

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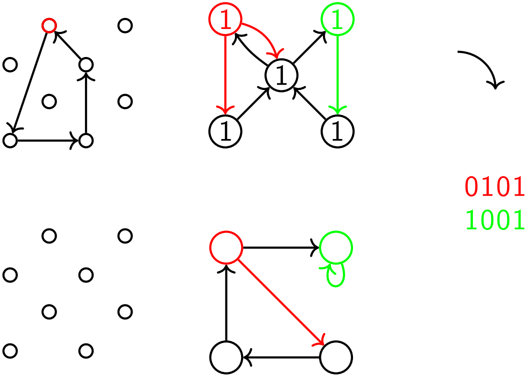

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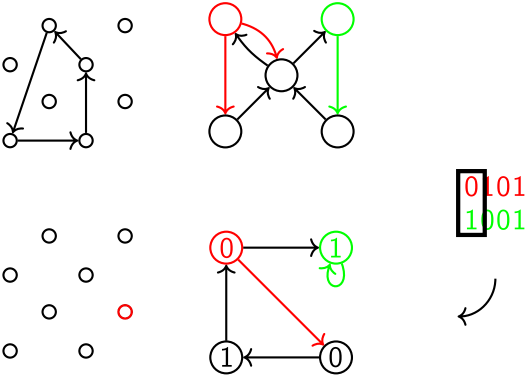

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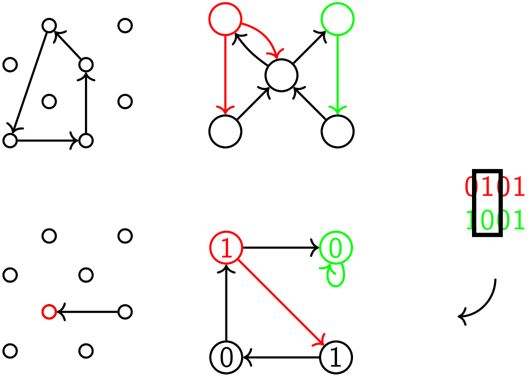

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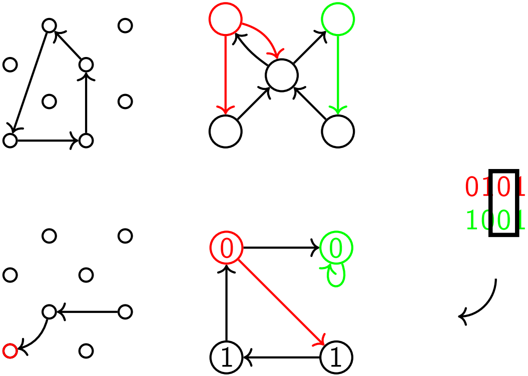

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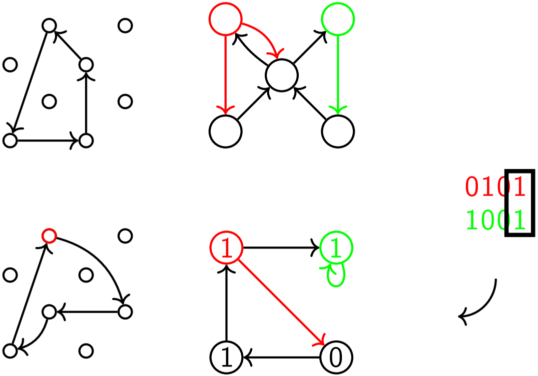

Using output functions to get attractors

$$O = \neg\alpha_2 \vee \alpha_3$$

$$\alpha_3 \, \alpha_2 \, \alpha_1 \;\; O$$

$$0 \quad 0 \quad 0 \quad 1 \quad 1 \quad \underline{0 \quad 1 \quad 1} \qquad \Rightarrow \text{3-cycle}$$

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$$0 \quad 1 \quad 0$$

Using output functions to get attractors

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0 0 0 1 1 0 1 1 $\Rightarrow$ 3-cycle

0 1 0 0 1

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0 1 0 0 1

1 1 1

Using output functions to get attractors

$$O = \neg \alpha_2 \vee \alpha_3$$

0 0 0 1 1 0 1 1 $\Rightarrow$ 3-cycle

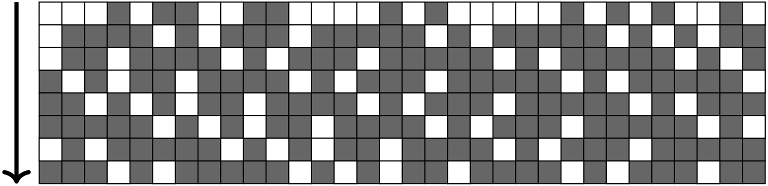
0 1 0 0 1

1 1 1 1 $\Rightarrow$ fixed point

From output functions to cellular automata

$$O = \neg \alpha_2 \vee \alpha_3$$

$$c_x \Leftarrow \neg c_{x-2} \vee c_{x-3}$$



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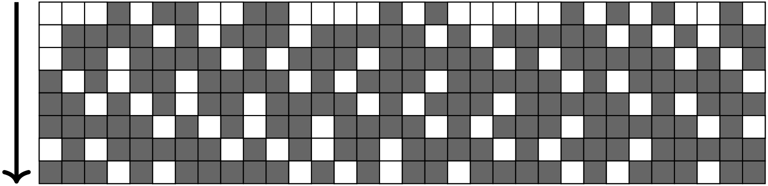


Theorem

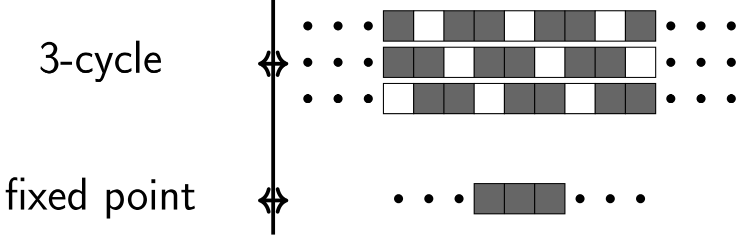
The fixed points of this cellular automata correspond one-to-one with the limit dynamics implied by the output function.

From output functions to cellular automata

Theorem

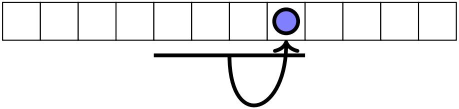
The fixed points of this cellular automata correspond one-to-one with the limit dynamics implied by the output function.

$$O = \neg\alpha_2 \vee \alpha_3 \Leftrightarrow c_x \Leftarrow \neg c_{x-2} \vee c_{x-3}$$



About *strictly* one-way cellular automata

$$c_x \Leftarrow \neg c_{x\underline{-2}} \vee c_{x\underline{-3}}$$



one-way cellular automaton

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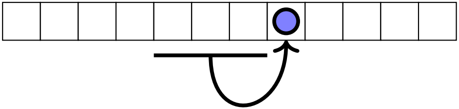

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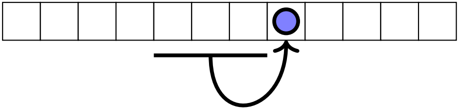
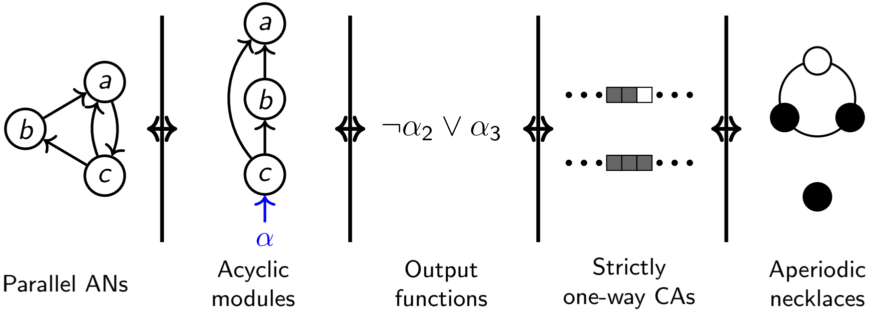


strictly one-way cellular automaton

Theorem

All strictly one-way cellular automata correspond to some automata network, i.e. their fixed points are equivalent to the networks' attractors.

The full picture



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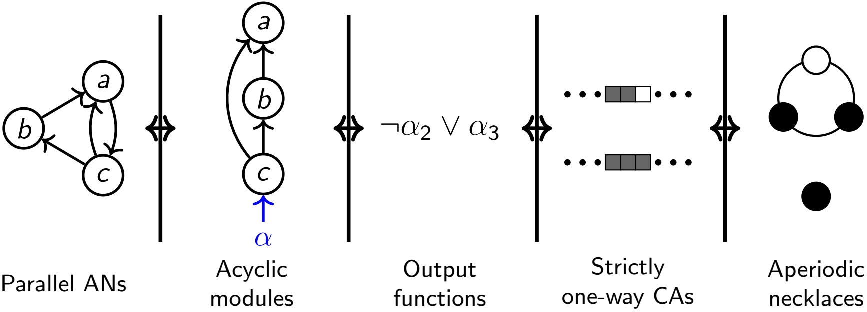

Thank you! We are now open for questions.