# State-based opacity of real-time automata

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# Content

# Background

- Opacity is a confidentiality property (firstly proposed by (Mazaré, 2004)) used to characterize information flow security, and has been widely used to describe all kinds of scenarios in security/privacy problems.

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- It describes whether a labeled (aka partially-observed) system can forbid an external intruder from making sure whether some secrets have been visited, given that the intruder knows complete knowledge of the system's structure but can only see outputs generated by the system.

# A general framework for opacity

Run-based opacity (Bryans et al., 2008)

$$q_0 \xrightarrow{e_1} q_1 \xrightarrow{e_2} \cdots \xrightarrow{e_n} q_n \qquad (\forall \text{ secret run})$$

$$q'_0 \xrightarrow{e'_1} q'_1 \xrightarrow{e'_2} \cdots \xrightarrow{e'_m} q'_m \qquad (\exists \text{ non-secret run})$$

$$\text{s.t. } \ell(e_1 \dots e_n) = \ell(e'_1 \dots e'_m) \qquad (\text{the same label seq.})$$

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## Language-based (aka trace-based) opacity

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## Verification results in untimed automata

- undecidable in labeled finite automata (LFAs) with $\epsilon$-labeling functions (Bryans et al., 2008)
- EXPTIME in LFAs when secret languages and non-secrete languages are regular (Lin, 2011)

# Two special classes of opacity: II

**State-based opacity (specified according to the time when secrets visited)**

$$q_0 \xrightarrow{e_1} q_1 \xrightarrow{e_2} \cdots \xrightarrow{e_i} q_i \xrightarrow{e_{i+1}} \cdots \xrightarrow{e_n} q_n \qquad (\forall \text{ secret state})$$

$$q'_0 \xrightarrow{e'_1} q'_1 \xrightarrow{e'_2} \cdots \xrightarrow{e'_j} q'_j \xrightarrow{e'_{j+1}} \cdots \xrightarrow{e'_m} q'_m \qquad (\exists \text{ non-secret state})$$

$$\text{s.t. } \ell(e_1 \dots e_i) = \ell(e'_1 \dots e'_j) =: \gamma_1$$

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## Verification results in untimed automata (plenty of)

Initial-state opacity (ISO) ($i = j = 0$), current-state opacity (CSO, $i = n, j = m$), infinite-step opacity (InfSO), and $K$-step opacity (KSO, $|\gamma_2| \leq K$) are PSPACE-complete in LFAs, and equivalent (Saboori and Hadjicostis, 2013) (Cassez, Dubreil, and Marchand, 2009) (Wu and Lafortune, 2013).

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- ISO is decidable in real-time automata (Wang, Zhan, and An, 2018).
- ISO, CSO, KSO, InfSO in real-time automata with complexity upper bounds on verification (Zhang, 2021)

# Content

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- $\ell : E \to \Sigma \cup \{\epsilon\}$ is the labeling function.

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- A run is either $\epsilon$ or a sequence $q_0 \xrightarrow{e_1/t_1} q_1 \xrightarrow{e_2/t_2} \cdots \xrightarrow{e_n/t_n} q_n =: \pi$, where $n \in \mathbb{Z}_+$, $(q_{i-1}, e_i, q_i) \in \Delta$, $t_i \in \mu(e_i)_{q_{i-1}q_i}$ for all $i \in [\![1, n]\!]$.

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- The timed word of $\pi$ is defined by $\tau(\pi) = (e_1, t_1')(e_2, t_2') \dots (e_n, t_n')$, where $t_i' = \sum_{k=1}^{i} t_k$ for all $i \in [\![1, n]\!]$.

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- A run is either $\epsilon$ or a sequence $q_0 \xrightarrow{e_1/t_1} q_1 \xrightarrow{e_2/t_2} \cdots \xrightarrow{e_n/t_n} q_n =: \pi$, where $n \in \mathbb{Z}_+$, $(q_{i-1}, e_i, q_i) \in \Delta$, $t_i \in \mu(e_i)_{q_{i-1}q_i}$ for all $i \in [\![1, n]\!]$.
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- The weight $\mathrm{WT}_\pi$ of $\pi$ is defined by $t'_n$.

## Example 1

Consider the RTA $\mathcal{A}_1$:

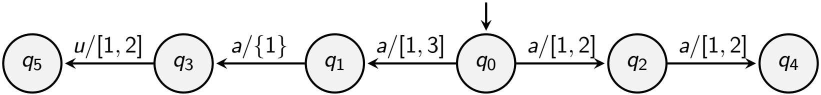


Figure 1: An RTA $\mathcal{A}_1$, $q_0$ is the initial state, $a$ is an observable event, $\ell(a) = a$, $u$ is unobservable, $\ell(u) = \epsilon$.

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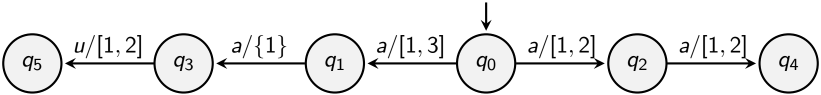


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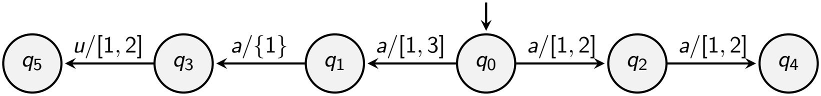


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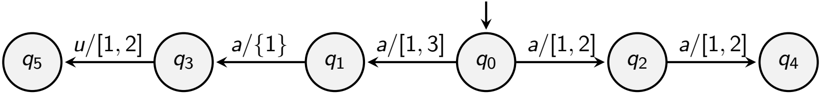


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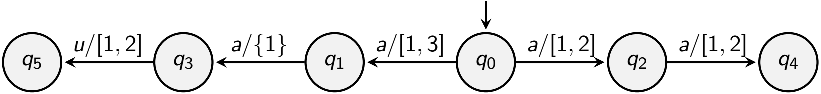


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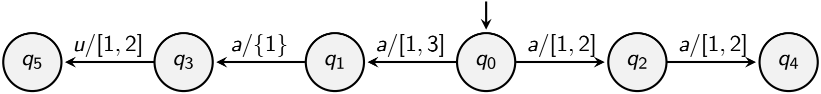


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\mathrm{WT}_\pi = 4, &\qquad \text{(weight)}\\
\ell(\tau(\pi)) = (a,2)(a,3). &\qquad \text{(timed label seq.)}
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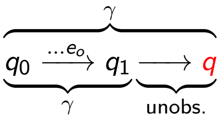

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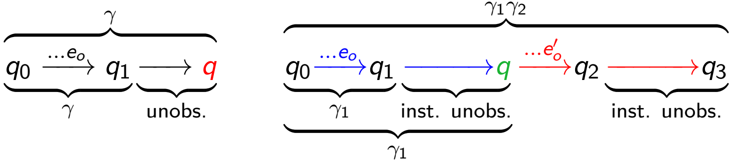
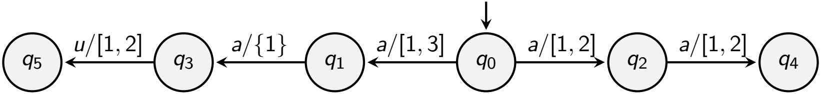
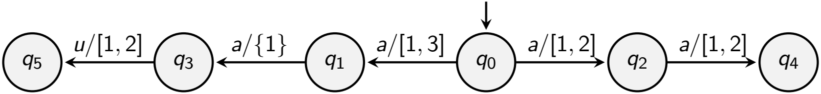
- Given $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^*$, $[\gamma]$ denotes the set of runs $\pi$ of $\mathcal{A}$ starting from initial states such that $\ell(\tau(\pi)) = \gamma$. $\mathrm{last}([\gamma])$
- $\mathrm{interm}(\gamma_1, \gamma_2) = \{q \in Q | (\exists \text{ runs } \pi_1, \pi_2)[(\mathrm{init}(\pi_1) \in Q_0) \wedge (\mathrm{last}(\pi_1) = \mathrm{init}(\pi_2) = q) \wedge (\ell(\tau(\pi_1)) = \gamma_1) \wedge (\ell(\tau(\pi_1\pi_2)) = \gamma_1\gamma_2) \wedge (\mathrm{WT}_{\pi_1} = \mathrm{last}_R(\gamma_1)) \wedge (\mathrm{WT}_{\pi_2} = \mathrm{last}_R(\gamma_2) - \mathrm{last}_R(\gamma_1))]\}$: the set of states $\mathcal{A}$ can be in when $\mathcal{A}$ has just generated timed label seq. $\gamma_1$, given that the current timed label seq. is $\gamma_1\gamma_2 \in (\Sigma \times \mathbb{R}_{\geq 0})^*$.



## Example 2 (cont. $\mathcal{A}_1$)

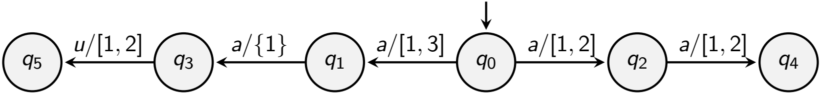


## Example 2 (cont. $\mathcal{A}_1$)




$$\text{last}([(a,2)]) = \{q_1, q_2\},$$

## Example 2 (cont. $\mathcal{A}_1$)




$$\text{last}([(a,2)]) = \{q_1, q_2\},$$
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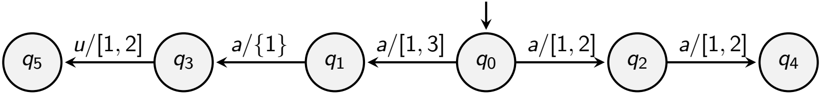
### Example 2 (cont. $\mathcal{A}_1$)




$$\text{last}([(a,2)]) = \{q_1, q_2\},$$
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# Current-state estimate

For $\mathcal{A}$, $x \subset Q$, and $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^*$, the current-state estimate is

$$\mathcal{M}(\mathcal{A}, \gamma|x) := \{q \in Q | (\exists q_0 \in x)(\exists n \in \mathbb{N})(\exists m \in \mathbb{N}) \left(\exists \text{ a run } \pi = q_0 \xrightarrow{e_1/t_1} \cdots \xrightarrow{e_n/t_n} q_n \xrightarrow{e_{n+1}/0} \cdots \xrightarrow{e_{n+m}/0} q\right) [(e_n \in E_o) \wedge (e_{n+1} \ldots e_{n+m} \in (E_{uo})^*) \wedge \ell(\tau(\pi)) = \gamma]\}.$$

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$\mathcal{M}(\mathcal{A}, \gamma)$ denotes the set of states $\mathcal{A}$ can be in when $\gamma$ has been generated.

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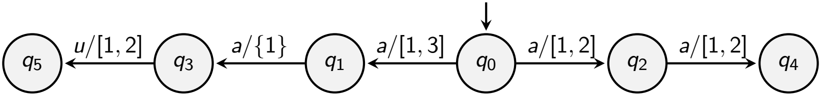
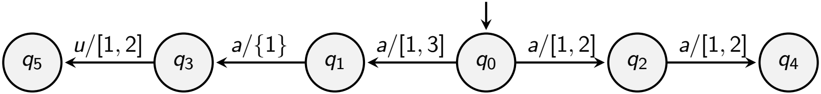
$\mathcal{M}(\mathcal{A}, \gamma | Q_0) =: \mathcal{M}(\mathcal{A}, \gamma) \subset \text{last}([\gamma])$ for $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^*$.

## Example 3 (cont. $\mathcal{A}_1$)



## Example 3 (cont. $\mathcal{A}_1$)

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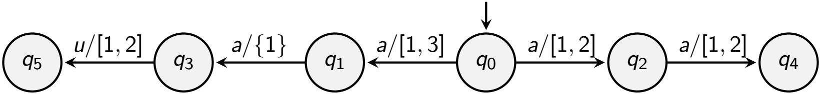


$$\text{last}([(a,2)]) = \{q_1, q_2\},$$
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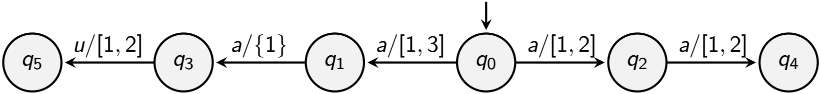
## Example 3 (cont. $\mathcal{A}_1$)

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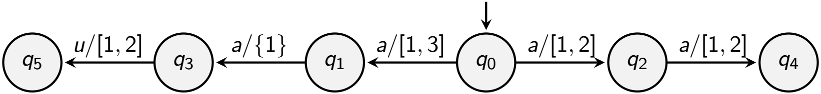
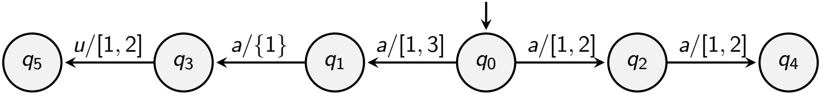



$$\begin{aligned} \mathrm{last}([(a,2)]) &= \{q_1, q_2\}, \\ \mathcal{M}(\mathcal{A}_1, (a,2)) &= \{q_1, q_2\}, \\ \mathrm{last}([(a,2)(a,3)]) &= \{q_3, q_4, q_5\}, \\ \mathcal{M}(\mathcal{A}_1, (a,2)(a,3)) &= \{q_3, q_4\}, \end{aligned}$$

## Example 3 (cont. $\mathcal{A}_1$)




$$\begin{aligned}
\text{last}([(a,2)]) &= \{q_1, q_2\},\\
\mathcal{M}(\mathcal{A}_1, (a,2)) &= \{q_1, q_2\},\\
\text{last}([(a,2)(a,3)]) &= \{q_3, q_4, q_5\},\\
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\mathcal{M}(\mathcal{A}_1, (a,2)(a,3)) &\subsetneq \text{last}([(a,2)(a,3)]).
\end{aligned}$$

# Content

Language generated by RTA $\mathcal{A}$: $\mathcal{L}(\mathcal{A}) = \{\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^* | \mathcal{M}(\mathcal{A}, \gamma) \neq \emptyset\}$

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**Definition 4 (ISO)**

An RTA $\mathcal{A}$ is called initial-state opaque (ISO) w.r.t. $Q_S$ if for every $\gamma \in \mathcal{L}(\mathcal{A})$, $\text{init}([\gamma]) \not\subset Q_S$.

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ISO means that when observing a timed label sequence $\gamma \in \mathcal{L}(\mathcal{A})$, not all possible initial states are secret, so that one cannot make sure whether the initial state is secret.

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**Definition 5 (CSO)**

An RTA $\mathcal{A}$ is called current-state opaque (CSO) w.r.t. $Q_S$ if for every $\gamma \in \mathcal{L}(\mathcal{A})$, in $\mathcal{M}(\mathcal{A}, \gamma)$ there exists at least one non-eventually-secret state.

### Definition 6

A state $q$ of an RTA $\mathcal{A}$ is called eventually secret if either (1) $q$ is secret or (2) there is an unobservable path starting from $q$ and along each of such paths at least one secret state will be visited.

## Definition 6

A state $q$ of an RTA $\mathcal{A}$ is called eventually secret if either (1) $q$ is secret or (2) there is an unobservable path starting from $q$ and along each of such paths at least one secret state will be visited.

## Proposition 1

*A state $q$ is not eventually secret iff (1) $q \notin Q_S$ and (2) either there is no unobservable path from $q$ or there is an unobservable path from $q$ without any secret state that either ends at a dead state or contains a cycle.*

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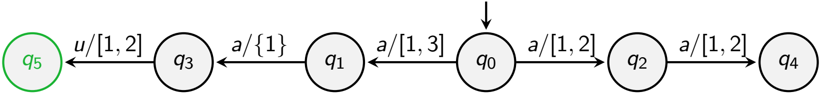
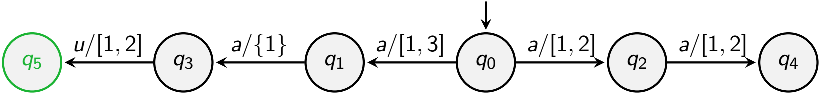
## Example 7 (cont. $\mathcal{A}_1$)



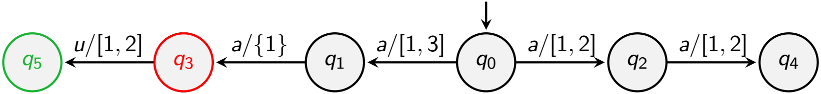

Let $Q_S = \{q_5\}$. Then $q_3$ is eventually secret because of the unique unobservable path $q_3 \xrightarrow{u} q_5$ (with $q_5$ dead, i.e., no transition starts at $q_5$).

Example 8 (cont. $\mathcal{A}_1$)



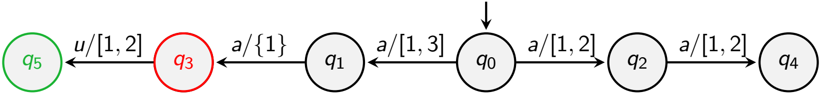

Let $Q_S = \{q_5\}$.

Example 8 (cont. $\mathcal{A}_1$)



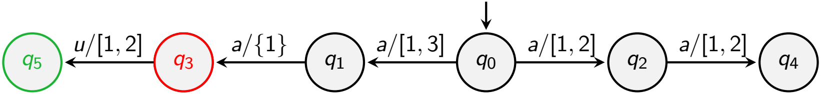

Let $Q_S = \{q_5\}$. In Example 7, we have shown $q_3$ is eventually secret.

Example 8 (cont. $\mathcal{A}_1$)




Let $Q_S = \{q_5\}$. In Example 7, we have shown $q_3$ is eventually secret. In addition, none of $q_1, q_0, q_2, q_4$ is eventually secret.

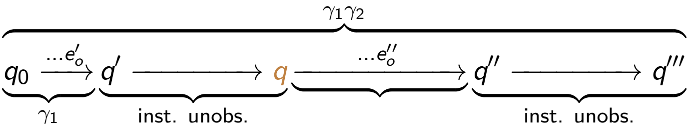
Example 8 (cont. $\mathcal{A}_1$)




Let $Q_S = \{q_5\}$. In Example 7, we have shown $q_3$ is eventually secret. In addition, none of $q_1, q_0, q_2, q_4$ is eventually secret. Hence $\mathcal{A}_1$ is not CSO w.r.t. $\{q_5\}$.

## Definition 9 (InfSO and KSO)

An RTA $\mathcal{A}$ is called infinite-step opaque (InfSO) w.r.t. $Q_S$ if for all $\gamma_1\gamma_2 \in \mathcal{L}(\mathcal{A})$ such that $1 \leq |\gamma_2|$, interm$(\gamma_1, \gamma_2)$ contains at least one non-secret state $q$.




(*)

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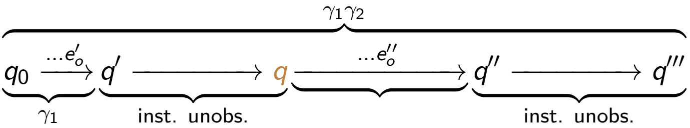



(*)

When observing $\gamma_1\gamma_2$ with $1 \le |\gamma_2|$, one cannot make sure whether the state when $\gamma_1$ has just been generated is secret.

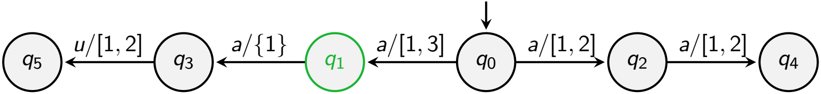
## Definition 9 (InfSO and KSO)

An RTA $\mathcal{A}$ is called infinite-step opaque (InfSO) ($K$-step opaque (KSO)) w.r.t. $Q_S$ if for all $\gamma_1\gamma_2 \in \mathcal{L}(\mathcal{A})$ such that $1 \le |\gamma_2| (\le K)$, $\mathrm{interm}(\gamma_1, \gamma_2)$ contains at least one non-secret state $q$.

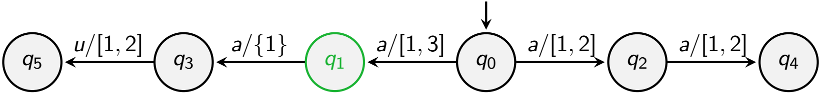
$$\overbrace{\underbrace{q_0 \xrightarrow{\dots e_o'}}_{\gamma_1} \underbrace{q' \longrightarrow q}_{\text{inst. unobs.}} \underbrace{\xrightarrow{\dots e_o''}}_{(\le K \text{ obs. events})} \underbrace{q'' \longrightarrow q'''}_{\text{inst. unobs.}}}^{\gamma_1\gamma_2} \qquad (*)$$

When observing $\gamma_1\gamma_2$ with $1 \le |\gamma_2| (\le K)$, one cannot make sure whether the state when $\gamma_1$ has just been generated is secret.

## Example 10 (cont. $\mathcal{A}_1$)



## Example 10 (cont. $\mathcal{A}_1$)




For $(a,3)(a,4)$, we only have

$$\overbrace{\underbrace{q_0 \xrightarrow{a/3}}_{\gamma_1=(a,3)} \underbrace{q_1 \xrightarrow{\quad\epsilon\quad} q_1}_{\text{unobs. secret}}}^{\gamma_1\gamma_2=(a,3)(a,4)} \xrightarrow{a/1} q_3 \implies \text{interm}(\mathcal{A}_1,(a,3),(a,4)) = \{q_1\}$$

which violates InfSO, i.e., $\mathcal{A}_1$ is not InfSO w.r.t. $\{q_1\}$.

## Definition 11

For an RTA $\mathcal{A}$, we define its pre-observer as a deterministic automaton

$$\mathcal{A}_{\text{obs}}^{\text{pre}} = (X, \Sigma \times \mathbb{R}_{\geq 0}, x_0, \delta_{\text{obs}}^{\text{pre}}), \tag{1}$$

where $X \subset 2^Q \setminus \{\emptyset\}$ is the state set, $\Sigma \times \mathbb{R}_{\geq 0}$ the (infinite) alphabet, $x_0 = \mathcal{M}(\mathcal{A}, \epsilon) \in X$ the unique initial state, $\delta_{\text{obs}}^{\text{pre}} \subset X \times (\Sigma \times \mathbb{R}_{\geq 0}) \times X$ the transition relation: for all $x, x' \in X$ and $(\sigma, t) \in \Sigma \times \mathbb{R}_{\geq 0}$, $(x, (\sigma, t), x') \in \delta_{\text{obs}}^{\text{pre}}$ iff $x' = \mathcal{M}(\mathcal{A}, (\sigma, t)|x)$. For all $x \subset Q$ different from $x_0$, $x \in X$ iff there is $\gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^+$ such that $x = \mathcal{M}(\mathcal{A}, \gamma)$.

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- After $\delta_{\text{obs}}^{\text{pre}}$ is recursively extended to $\delta_{\text{obs}}^{\text{pre}} \subset X \times (\Sigma \times \mathbb{R}_{\geq 0})^* \times X$, one has for all $x \in X$ and $(\sigma_1, t_1) \ldots (\sigma_n, t_n) =: \gamma \in (\Sigma \times \mathbb{R}_{\geq 0})^+$, $(x_0, \gamma, x) \in \delta_{\text{obs}}^{\text{pre}}$ iff $\mathcal{M}(\mathcal{A}, \tau(\gamma)) = x$, where $\tau(\gamma) = (\sigma_1, t_1)(\sigma_1, t_1 + t_2) \ldots (\sigma_n, t_1 + \cdots + t_n)$.

## Definition 11

For an RTA $\mathcal{A}$, we define its pre-observer as a deterministic automaton

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- Alphabet $\Sigma \times \mathbb{R}_{\geq 0}$ is not finite, one cannot compute the whole $\mathcal{A}_{\text{obs}}^{\text{pre}}$. Next, we define observer $\mathcal{A}_{\text{obs}}$ as a finite sub-automaton of $\mathcal{A}_{\text{obs}}^{\text{pre}}$.

## Definition 12

For an RTA $\mathcal{A}$, consider its pre-observer $\mathcal{A}_{\text{obs}}^{\text{pre}} = (X, \Sigma \times \mathbb{R}_{\geq 0}, x_0, \delta_{\text{obs}}^{\text{pre}})$, we define its observer as a finite automaton

$$\mathcal{A}_{\text{obs}} = (X, \Sigma_{\text{obs}}, x_0, \delta_{\text{obs}}), \tag{2}$$

where $\Sigma_{\text{obs}}$ (resp., $\delta_{\text{obs}}$) is a finite subset of $\Sigma \times \mathbb{Q}_{\geq 0}$ (resp., $\delta_{\text{obs}}^{\text{pre}}$), such that if there exists a transition from $x \in X$ to $x' \in X$ in $\delta_{\text{obs}}^{\text{pre}}$ then at least one such transition belongs to $\delta_{\text{obs}}$.

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where $\Sigma_{\text{obs}}$ (resp., $\delta_{\text{obs}}$) is a finite subset of $\Sigma \times \mathbb{Q}_{\geq 0}$ (resp., $\delta_{\text{obs}}^{\text{pre}}$), such that if there exists a transition from $x \in X$ to $x' \in X$ in $\delta_{\text{obs}}^{\text{pre}}$ then at least one such transition belongs to $\delta_{\text{obs}}$.

## Remark 1

## Definition 12

For an RTA $\mathcal{A}$, consider its pre-observer $\mathcal{A}_{\text{obs}}^{\text{pre}} = (X, \Sigma \times \mathbb{R}_{\geq 0}, x_0, \delta_{\text{obs}}^{\text{pre}})$, we define its observer as a finite automaton

$$\mathcal{A}_{\text{obs}} = (X, \Sigma_{\text{obs}}, x_0, \delta_{\text{obs}}), \tag{2}$$

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- *For an RTA $\mathcal{A}$, it may have more than one observer, because $\Sigma_{\text{obs}}$ may not be unique; but $X$ and $x_0$ must be unique.*

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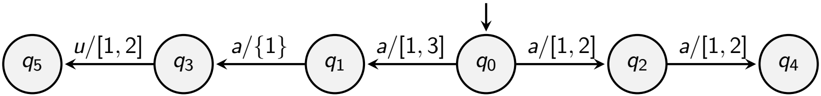
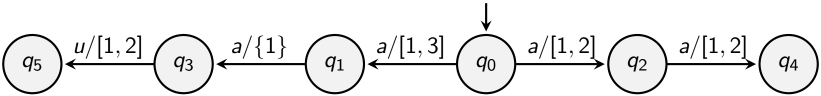
## Remark 1

- *For an RTA $\mathcal{A}$, it may have more than one observer, because $\Sigma_{\text{obs}}$ may not be unique; but $X$ and $x_0$ must be unique.*
- *For a labeled finite automaton, it has a unique observer, which is actually the powerset construction used for determinizing the automaton.*

## Example 13 (cont. $\mathcal{A}_1$)



## Example 13 (cont. $\mathcal{A}_1$)




One of its observers is

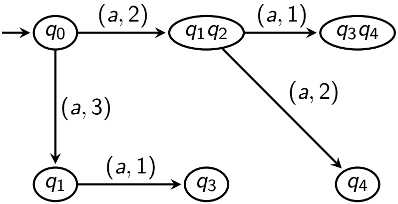



Figure 2: $\mathcal{A}_{1\,\text{obs}}$.

### Theorem 14

*An RTA $\mathcal{A}$ is CSO w.r.t. $Q_S$ iff in observer $\mathcal{A}_{\text{obs}}$, every reachable state $x$ contains at least one non-eventually-secret state of $\mathcal{A}$.*

## Theorem 14

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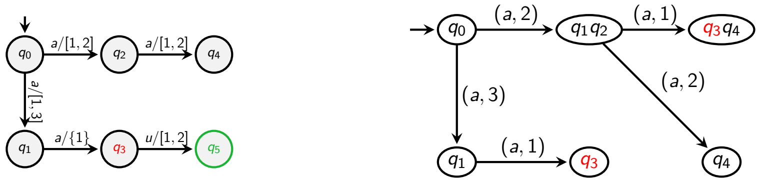
## Example 15 (cont. $\mathcal{A}_1$)




$\mathcal{A}_1$. $\mathcal{A}_{1\,\text{obs}}$.

Let $Q_S = \{q_5\}$, so the eventually secret states are $q_3$ and $q_5$.

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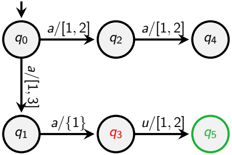
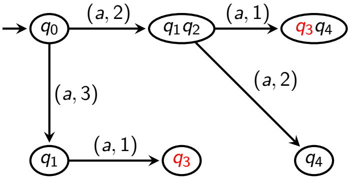
## Example 15 (cont. $\mathcal{A}_1$)




$\mathcal{A}_1$.




$\mathcal{A}_{1\,\mathrm{obs}}$.

Let $Q_S = \{q_5\}$, so the eventually secret states are $q_3$ and $q_5$. In $\mathcal{A}_{1\,\mathrm{obs}}$, there is a reachable state $\{q_3\}$ which only contains eventually secret states, then $\mathcal{A}_1$ is not CSO w.r.t. $\{q_5\}$.

**Theorem 16**

*For an RTA $\mathcal{A}$, its observer $\mathcal{A}_{\mathrm{obs}}$ can be computed in* 2-EXPTIME *in the size of $\mathcal{A}$.*

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### Proof sketch

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## Proof sketch

- Compute the initial state $x_0 = \mathcal{M}(\mathcal{A}, \epsilon)$ in polynomial time.

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- Compute the initial state $x_0 = \mathcal{M}(\mathcal{A}, \epsilon)$ in polynomial time.
- Starting from $x_0$, find all reachable states step by step together with the corresponding transitions: check for all $x_1, x_2 \subset Q$ and $\sigma \in \Sigma$, whether there is a transition $(x_1, (\sigma, t), x_2)$ for some $t \in \mathbb{Q}_{\geq 0}$ (exponentially many times, each in doubly exponential time).

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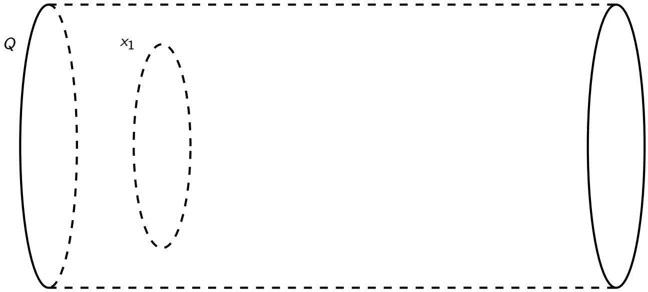
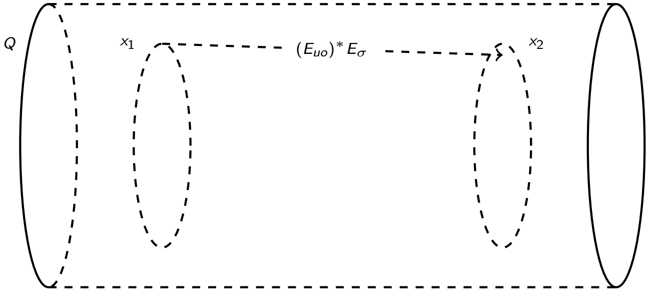
- Starting from $x_0$, find all reachable states step by step together with the corresponding transitions: check for all $x_1, x_2 \subset Q$ and $\sigma \in \Sigma$, whether there is a transition $(x_1, (\sigma, t), x_2)$ for some $t \in \mathbb{Q}_{\geq 0}$ (exponentially many times, each in doubly exponential time).
- In addition, for all $x_1, x_2, x_3 \subset Q$, if we find two transitions $(x_1, (\sigma, t), x_2)$ and $(x_1, (\sigma, t'), x_3)$ for some $t, t' \in \mathbb{Q}_{\geq 0}$, then $x_2 \subset x_3$ implies $x_3 \not\subset \mathcal{M}(\mathcal{A}, (\sigma, t)|x_1)$. This guarantees that if there exists a transition from $x_1 \subset Q$ to $x_2 \subset Q$ in $\mathcal{A}_{\text{obs}}^{\text{pre}}$, then there also exists a transition from $x_1 \subset Q$ to $x_2 \subset Q$ in $\mathcal{A}_{\text{obs}}$.


Q


Q
$x_1$

$\sigma \in \Sigma,\ E_\sigma = \{e \in E | \ell(e) = \sigma\}$



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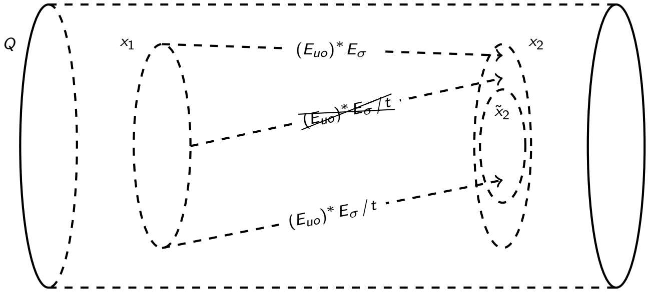
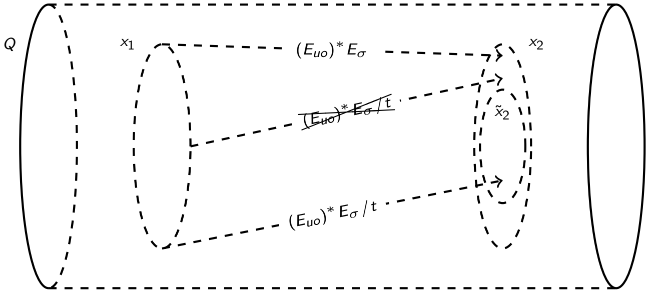



$t$ depends on $x_1, x_2, \tilde{x}_2$

$$x_1 \ni q_1(\exists) \xrightarrow{(E_{uo})^* E_\sigma / t} (\forall) q_2 \in \tilde{x}_2 (\forall\exists)$$

$$x_1 \ni q_1(\forall) \not\xrightarrow{(E_{uo})^* E_\sigma / t} (\forall) q_2 \in x_2 \setminus \tilde{x}_2 (\forall\forall)$$

$\sigma \in \Sigma$, $E_\sigma = \{e \in E | \ell(e) = \sigma\}$

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$$\Longrightarrow x_1 \xrightarrow{(\sigma, t)} \mathcal{M}(\mathcal{A}, \epsilon | \tilde{x}_2) \text{ a transition of } \mathcal{A}_{\text{obs}}$$

# Further reading for computing $\mathcal{A}_{\text{obs}}$

- NP-complete exact path length problem in weighted directed graphs $(\mathbb{Q}^k, V, A)$[1]

[1] M. Nykänen and E. Ukkonen (2002). "The exact path length problem". In: *Journal of Algorithms* 42.1, pp. 41–53.

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- Presburger arithmetic[3]
- Observer of labeled weighted automata over the monoid $(\mathbb{Q}^k, +)$, computable in 2-EXPTIME[4]

[1] M. Nykänen and E. Ukkonen (2002). "The exact path length problem". In: *Journal of Algorithms* 42.1, pp. 41–53.

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# Content

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## An open question

Lower bounds on verification of state-based opacity in RTAs (EXPSPACE or 2-EXPTIME?)

Thank you for your attention!

Questions or comments?

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