Fixed Point Constructions in Tilings and Cellular Automata

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Definitions & construction

Fault-tolerant CA

- Can a 1D CA remember information indefinitely, if cells make random errors independently with rate $\epsilon > 0$?
- [Kurdyumov 1978, Gács 1986, Gács 2001]: Yes!
- Introduces fixed point construction for CA
- Uses other tools and tricks, very complex

Durand-Romashchenko-Shen papers

- Extract essence of fixed point construction from Gács's work
- [DRS 2008, DRS 2009a, DRS 2009b, DRS 2010, DRS 2012]
- Sofic realization: set of rows projects to any effectively closed 1D subshift (improvement over [Hochman 2009], independently by [Aubrun-Sablik 2013])
- [Durand-Romashchenko 2021]: minimal sets of rows realizable by minimal SFT

Durand-Romashchenko-Shen papers

- Complex tilings: minimum Kolmogorov complexity of $n \times n$ pattern is $\Omega(n)$ (reproving [Durand-Levin-Shen 2008])
- Robust tilings: random subset of $\mathbb{Z}^2$ can break local rules without destroying global structure
- Complex robust tilings

Seas of squares

- [Westrick 2017]: Results on binary subshift where 1s form squares
- All effective subshifts with distinct-size squares are sofic
- Restriction of sizes to any r.e. set is sofic
- [Westrick 2019]: Preprint on topologically completely positive entropy

Unique ergodicity

- A CA with quiescent state q is uniquely ergodic if for every cell, the proportion of time steps it has non-q state approaches 0
- [Törmä 2015]: there exists a uniquely ergodic CA that is not nilpotent

Topological entropy

- Topological entropy of subshift: growth rate of number of distinct $[0, n-1]^d$-shaped patterns
- [Durand-Romashchenko 2021, Gangloff-Sablik 2017]: characterize entropies of transitive 2D SFTs (nontransitive case in [Hochman-Meyerovitch 2010])
- Top. entropy of CA: limit of entropies of traces
- [Hochman 2009, Guillon-Zinoviadis 2013]: characterize entropies of CA of every dimension

Expansive directions

- Expansive direction: thick line whose contents determine rest of tiling
- [Boyle-Lind 1997, Hochman 2011]: sets of nonexpansive directions of subshifts = closed sets of directions
- [Guillon-Zinoviadis]: sets of nonexpansive directions of SFTs = eff. closed sets of directions
- Fixed point construction for reversible CA